AP Calculus AB
Review 4.3 and 4.4

Name:

Find the derivatives of the following functions.

1. $y=\tan^{-1}(2x^3)$

$y' = \frac{1}{1+(2x^3)^2} \cdot 6x^2$

$= \frac{6x^2}{1+4x^6}$

2. $y=\sin^{-1}(3x+2)$

$y' = \frac{1}{\sqrt{1-(3x+2)^2}} \cdot 3$

$= \frac{3}{\sqrt{-9x^2-12x-3}}$

3. $y=\cot^{-1}(-3x^3)$

$y' = \frac{-1}{1+(-3x^3)^2} \cdot -9x^2$

$= \frac{9x^2}{1+9x^6}$

4. $y=\sec^{-1}(5x^5)$

$y' = \frac{1}{|5x^5|\sqrt{(5x^5)^2-1}} \cdot 25x^4$

$= \frac{1}{|5x^5|\sqrt{25x^{10}-1}} \cdot 25x^4$

SIDE NOTE: Are | | bars necessary in $\frac{d}{dx}\sec^{-1}(5x^4)$?

$= \frac{5}{|x|\sqrt{25x^{10}-1}}$

necessary!!

5. $y=\cos^{-1}\left(\frac{4}{x}\right), x>0$

$y' = \frac{-1}{\sqrt{1-(\frac{4}{x})^2}} \cdot -4x^{-2}$

$= \frac{4}{x^2\sqrt{\frac{x^2-16}{x^2}}}$

$= \frac{4}{x\sqrt{x^2-16}}$

How interesting!

6. $y=\sec^{-1}\left(\frac{x}{4}\right), x>0$

$y' = \frac{1}{|\frac{x}{4}|\sqrt{(\frac{x}{4})^2-1}} \cdot \frac{1}{4}$

$= \frac{1}{|\frac{x}{4}|\sqrt{\frac{x^2-16}{16}}} \cdot \frac{1}{4}$

$= \frac{1}{4 \cdot x \cdot \frac{1}{4} \cdot \frac{1}{4}\sqrt{x^2-16}} = \frac{4}{x\sqrt{x^2-16}}$

There are several ways to think about this

For each problem, $g(x) = f^{-1}(x)$ and $f(a) = b$. Find b and $g'(b)$.

7. $f(x) = 4x - 3,\ a = -3$

$f(-3) = -15$

$f'(x) = 4 = f'(-3)$

$g(-15) = -3$

$g'(-15) = \frac{1}{4}$

8. $f(x) = 2x^2 + 8,\ a = 4$

$f(4) = 40$

$f'(x) = 4x$

$f'(4) = 16$

$g(40) = 4$

$g'(40) = \frac{1}{16}$

9. $f(x) = -3\cos 4x + 5,\ a = \frac{\pi}{8}$

$f(\frac{\pi}{8}) = -3\cos\frac{\pi}{2} + 5 = 5$

$f'(x) = 12\sin 4x$

$f'(\pi/8) = 12$

$g(5) = \frac{\pi}{8}$

$g'(5) = \frac{1}{12}$

10. $f(x) = \sqrt{3x - 4},\ a = 3$

$f(3) = \sqrt{5}$

$f'(x) = \frac{1}{2}(3x-4)^{-1/2} \cdot 3$

$f'(3) = \frac{3}{2\sqrt{5}}$

$g(\sqrt{5}) = 3$

$g'(\sqrt{5}) = \frac{2\sqrt{5}}{3}$

11. Let $g(x) = f^{-1}(x)$. The line $y - 8 = \frac{3}{7}(x+2)$ is tangent to $f(x)$. Write an equation of a line tangent to $g(x)$.

$y + 2 = \frac{7}{3}(x - 8)$

Find the derivative of the following functions.

12. $y = e^{4\sin x}$

$y' = e^{4\sin x} \cdot 4\cos x$

13. $y = \ln x^4$

$y' = \frac{1}{x^4} \cdot 4x^3$

$= \frac{4}{x}$

14. $y = \log_3 \cos^{-1} x$

$y' = \frac{1}{\ln 3 \cos^{-1} x} \cdot \frac{-1}{\sqrt{1-x^2}}$

15. $y = 3^{4x^5}$

$y' = 3^{4x^5} \cdot \ln 3 \cdot 20x^4$

Let's get crazy! Find the derivative of the following functions.

16. $y = x^3e^x - (x+1)e^x$

$$y' = x^3 \cdot e^x + e^x \cdot 3x^2 - \left[(x+1)e^x + e^x \cdot 1\right]$$

$$= x^3e^x + 3x^2e^x - (x+1)e^x - e^x$$

$$= e^x\left[x^3 + 3x^2 - x - 1 - 1\right]$$

$$= e^x\left[x^3 + 3x^2 - x - 2\right]$$

17. $y = (3x^4 + 5)\ln\left(2x^3\right)$

$$y' = (3x^4 + 5) \cdot \frac{1}{2x^3} \cdot 6x^2 + \ln(2x^3) \cdot 12x^3$$

$$= (3x^4 + 5) \cdot \frac{3}{x} + 12x^3\ln(2x^3)$$

$$= 9x^3 + \frac{15}{x} + 12x^3\ln(2x^3)$$

18. $y = \ln\cos e^{x^4}$

$$y' = \frac{1}{\cos e^{x^4}} \cdot -\sin e^{x^4} \cdot e^{x^4} \cdot 4x^3$$

$$= \frac{-4x^3e^{x^4}\sin e^{x^4}}{\cos e^{x^4}}$$

19. $y = \left(\log_5 \sin^{-1}(4x+7)\right)^6$

$$y' = 6\left[\log_5 \sin^{-1}(4x+7)\right]^5 \cdot \frac{1}{\ln 5 \sin^{-1}(4x+7)} \cdot \frac{1}{\sqrt{1-(4x+7)^2}} \cdot 4$$

This is good practice.
Nothing this tough on the quiz,
but if you can do, you're great!

Extensions

20. At what point on the graph of $y = 3e^x - 1$ is the tangent line parallel to the line $y = \frac{1}{4}x - 5$?

$y' = 3e^x = \frac{1}{4}$

$e^x = \frac{1}{12}$

$x = \ln \frac{1}{12}$

$y = 3e^{\ln 1/12} - 1$

$= 3 \cdot 1/12 - 1$

$= 1/4 - 1$

$= -3/4$

$\left(\ln \frac{1}{12}, \frac{-3}{4}\right)$

21. The total number of people who received a text message after t minutes can be modeled by the function:

$$P(t) = \frac{150}{1 + e^{3-t}}$$

a. Calculate the initial number of people who received the text message.

$P(0) = \frac{150}{1+e^3} \approx 7$ people

b. How fast is the text message being distributed after 5 minutes?

$P'(t) = \frac{(1+e^{3-t}) \cdot 0 - 150e^{3-t} \cdot -1}{(1+e^{3-t})^2}$

$= \frac{150e^{3-t}}{(1+e^{3-t})^2}$

$P'(5) = \frac{150e^{-2}}{(1+e^{-2})^2} = 15.7490$ $\frac{\text{people}}{\text{min}}$

22. Write the equation of the tangent line to $y = \sin^{-1}\left(\frac{x}{5}\right)$ at $x = 2$.

$y' = \frac{1}{\sqrt{1 - \frac{x^2}{25}}} \cdot \frac{1}{5} = \frac{1}{5\sqrt{\frac{25-x^2}{25}}} = \frac{1}{\sqrt{25-x^2}}$

$y(2) = 0.4115$

$y'(2) = 1/\sqrt{21}$

$y - 0.4115 = \frac{1}{\sqrt{21}}(x-2)$

23. What is the domain of the derivative of $y = \log_4\left(\frac{1}{x}\right)$? $\rightarrow x > 0$

$y' = \frac{1}{\ln 4 \cdot \frac{1}{x}} \cdot -1x^{-2} = \frac{-1 \cdot x}{\ln 4 \, x^2} = \frac{-1}{\ln 4 \cdot x}$

$x > 0$

24. A line with slope m passes through the origin and is tangent to $y = \ln(5x)$. What is the value of m?

$y' = \frac{5}{5x} = \frac{1}{x} = m$

slope of tangent line

$y = mx$

$\ln(5x) = \frac{1}{x} \cdot x$

$e^1 = 5x$

$x = \frac{e}{5}$

$m = \frac{1}{x} = \frac{5}{e}$