AP Calculus AB **Name:**
Review 4.1, 4.2

Find the derivative of the following equations. Please make sure to factor and simplify.

1. $y = \sin 8x$

$y' = 8\cos 8x$

2. $y = (6-11x)^{-3}$

$y' = -3(6-11x)^{-4} \cdot -11$

$= 33(6-11x)^{-4}$

3. $y = \tan(\csc x)$

$y' = \sec^2(\csc x) \cdot -\csc x \cot x$

4. $y = 7\cot\left(\frac{5}{x}\right)$

$y' = -7\csc^2\left(\frac{5}{x}\right) \cdot -5x^{-2}$

$= \dfrac{35\csc^2\left(\frac{5}{x}\right)}{x^2}$

5. $y = \sin^5(4x-1)$

$y' = 5\sin^4(4x-1)\cos(4x-1) \cdot 4$

$= 20\sin^4(4x-1)\cos(4x-1)$

6. $y = \dfrac{2x}{\sqrt{16-x^4}}$

$y' = (16-x^4)^{1/2} \cdot 2 - \cancel{2}x \cdot \frac{1}{\cancel{2}}(16-x^4)^{-1/2} \cdot -4x^3 \;/\; 16-x^4$

$= 2(16-x^4)^{-1/2}\left[(16-x^4)^{2/2} + 2x^4\right] \;/\; 16-x^4$

$= 2(16-x^4)^{-1/2}(16-x^4+2x^4) \;/\; 16-x^4$

$= \dfrac{2x^4+32}{(16-x^4)^{3/2}}$

Use the table to answer questions 7-9.

x	f(x)	g(x)	f'(x)	g'(x)
-1	8	2	2	5
8	7	3	-1	11

7. Find $\frac{d}{dx}h(x)$ at $x = \text{-}1$ if $h(x) = g(f(x))$.

$h'(x) = g'(f(x)) \cdot f'(x)$

$h'(-1) = g'(f(-1))f'(-1) = g'(8) \cdot 2 = 22$

8. Find $\frac{d}{dx}m(x)$ at $x = \text{-}1$ if $m(x) = 4x^2 \cdot (g(x))^2$.

$m'(x) = 4x^2 \cdot 2g(x) \cdot g'(x) + [g(x)]^2 \cdot 8x$

$m'(-1) = 4 \cdot 2 \cdot 2 \cdot 5 + 4 \cdot 8 \cdot -1$

$= 80 - 32 = 48$

9. Find $\frac{d}{dx}a(x)$ at $x = 8$ if $a(x) = [f(x) + g(x)]^3$.

$a'(x) = 3(f(x) + g(x))^2 \cdot (f'(x) + g'(x))$

$a'(8) = 3(10)^2 \cdot 10$

$= 3000$

10. Write the equation of the tangent line at (-4,3) if $\frac{dy}{dx} = 5x - 2y$.

$$\left.\frac{dy}{dx}\right|_{(-4,3)} = 5 \cdot -4 - 2 \cdot 3 = -26$$

$$y - 3 = -26(x+4)$$

11. Find $\frac{dy}{dx}$ for $2x - xy + 4y = 9$.

$$2\frac{\cancel{dx}}{\cancel{dx}} - x \cdot \frac{dy}{dx} + y \cdot -1\frac{\cancel{dx}}{\cancel{dx}} + 4\frac{dy}{dx} = 0\frac{\cancel{dx}}{\cancel{dx}}$$

$$\frac{dy}{dx}(-x+4) = y - 2$$

$$\frac{dy}{dx} = \frac{y-2}{4-x}$$

12. Show that $\frac{d^2y}{dx^2} = \frac{-16}{y^3}$ for $x^2 + y^2 = 16$.

$$2x\frac{\cancel{dx}}{\cancel{dx}} + 2y\frac{dy}{dx} = 0\frac{\cancel{dx}}{\cancel{dx}}$$

$$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$$

$$\frac{d^2y}{dx^2} = \frac{y \cdot -1\frac{dx}{dx} - -x \cdot 1\frac{dy}{dx}}{y^2}$$

$$= \frac{-y + x \cdot \frac{-x}{y}}{y^2}$$

$$= \frac{-y^2 - x^2}{y^3}$$

$$= \frac{-16}{y^3}$$

* since

$x^2 + y^2 = 16$

$-x^2 - y^2 = -16$