Key

Alg 2 Trig H Series and Sequences Study Guide

1. Write a general term (a_n) for the sequence:

$\frac{2}{4}\ \frac{3}{5}\ \frac{4}{5}\ \frac{5}{7}$

a. $\frac{3}{4},\frac{4}{5},\frac{5}{6},\ldots$ $\boxed{a_n=\frac{n+2}{n+3}}$

$n=1,2,3$

b. $3,-15,75,-375$

$\boxed{a_n=(-1)^{n+1}\cdot 3(5)^{n-1}}$

c. $\frac{2}{3},\frac{3}{4},\frac{4}{5},\frac{5}{6},\ldots$ $\boxed{a_n=\frac{n+1}{n+2}}$

$n=1\ 2\ 3\ 4$

2. Write each series using summation notation with the summing index k starting at k = 1.

$n=1\quad 2\quad 3\quad 4$

a. $-9+16-25+36$

$\boxed{\sum_{k=1}^{4}(-1)^k(k+2)^2}$

b. $\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\frac{1}{2^5}+\frac{1}{2^6}$

$\boxed{\sum_{k=1}^{5}\frac{1}{2^{k+1}}}$

c. $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\frac{1}{7}$

$\boxed{\sum_{=1}^{7}(-1)^{k+1}\frac{1}{k}}$

3. Given the following sequences, write both the EXPLICIT and RECURSIVE formulas.

a. $-12,-4,4,12,20,\ldots.$

EXP: $a_n=-12+(n-1)8$

REC: $a_n=a_{n-1}+8,\ n\geq 2$ integers

$a_1=-12$

b. $\frac{3}{4},3,12,48,\ldots$

EXP: $a_n=\frac{3}{4}(4)^{n-1}$

REC $=$ $a_n=a_{n-1}\cdot 4,\ n\geq 2$ integers

$a_1=3/4$

c. $2,-10,50,-250$

EXP: $a_n=2(-5)^{n-1}$

REC: $a_n=a_{n-1}\cdot -5,\ n\geq 2$ integers

$a_1=2$

$a\ =4+-5(87-1)$

4. Find the 87th term in an arithmetic sequence with $a_1=4$ and d = - 5.

$a_{87}=4+-5(87-1)$

$\boxed{a_{87}=-426}$

5. Find the 15th term in a geometric sequence with $a_2 = 7$ and r = 2.

$a_{15} = \frac{7}{2}(2)^{15-1}$

$\boxed{a_{15} = 57{,}344}$

6. Find the 13th term of:

a) a geometric sequence with: $a_3 = 3$ and $a_7 = 768$

$768 = 3 \cdot r^4$

$256 = r^4$

$\pm 4 = r$

$a_1 = \frac{3}{16}$

so $a_{13} = \frac{3}{16}(\pm 4)^{12}$

$\boxed{a_{13} = 3{,}145{,}728}$

b) an arithmetic sequence with: $a_5 = -16$ and $a_{11} = 14$

$14 = a_1 + d(11-1)$

$-16 = a_1 + d(5-1)$

$d = 5 \quad a_1 = -36$

$a_{13} = -36 + 5(13-1)$

$= 24$

7. We studied infinite arithmetic and geometric series. Which converge and which diverge? Why?

ONLY $a_n = 0$

$0 < |r| < 1$

$|r| > 1$

8. Consider the following geometric series formulas. What is the difference and how do we get from the first to the second and why?

$$S_n = \frac{a_1(1 - r^n)}{1 - r}, \qquad S_\infty = \frac{a_1}{1 - r}$$

if $0 < |r| < 1$

as $n \to \infty$

this $(r^n) \to 0$

9. First determine if the series is arithmetic or geometric, or neither. Then, find the indicated sum.

a. $\sum_{k=1}^{8} 8\left(\frac{1}{4}\right)^{k}$ geometric

$S_8 = \frac{2(1-\frac{1}{4}^{8})}{1-\frac{1}{4}}$

$S_8 \approx 2.66$

b. $\sum_{k=1}^{4} (-1)^{k-1}(4k+1)$ neither

$5 - 9 + 13 - 17$

$\boxed{-8}$

c. $\sum_{k=1}^{25} (7-2k)$ arithmetic

$S_{25} = \frac{25}{2}(5 + -43)$

$\boxed{S_{25} = -475}$

d. $\sum_{k=1}^{\infty} \left(-\frac{1}{2}\right)^{k+1}$ infinite geometric

$S_\infty = \frac{\frac{1}{4}}{1+\frac{1}{2}} = \frac{1}{4}\cdot\frac{2}{3} = \boxed{\frac{1}{6}}$

10. Find the sum of ~~the positive~~ multiples of 3 from 4 to 99 (inclusive) arithmetic

$6, 9, 12, \ldots, 99$

$a_1 = 6$ $d = 3$

$a_n = 99$

$99 = 6 + 3(n-1)$

$n = 32$

$S_{32} = \frac{32}{2}(6 + 99)$

$\boxed{S_{32} = 1680}$

11. Find the sum of all even integers from 5 to 42.

$6, 8, 10, \ldots, 42$

$a_1 = 6$ $d = 2$

$a_n = 42$

$42 = 6 + (n-1)2$

$n = 19$

$S_{19} = \frac{19}{2}(6 + 42)$

$\boxed{S_{19} = 456}$

12. Careful here!!! ☺ Find: $\sum_{k=7}^{40} -3 + 2(k-1)$

$\sum_{k=1}^{40} - \sum_{k=1}^{6} \Rightarrow \frac{40}{2}(-3 + 75) - \frac{6}{2}(-3 + 7)$

$1440 - 12$

1428

13. Determine the summation of the following series:

a. 22 + 18 + 14 + . . . + -26

$\overset{-4}{\frown}\ \overset{-4}{\frown}$

$a_1 = 22$
$d = -4$

$-26 = 22 + -4(n-1)$
$-26 = 22 - 4n + 4$
$n = 13$

$S_{13} = \frac{13}{2}(22 - 26)$
$S_{13} = -26$

b. 28 + 14 + 7 + . . . + 0.875

$a_1 = 28$
$r = \frac{1}{2}$

$.875 = 28\left(\frac{1}{2}\right)^{n-1}$
$\frac{1}{32} = \frac{1}{2}^{n-1}$
$\left(\frac{1}{2}\right)^5 = \frac{1}{2}^{n-1}$
$n = 6$

$S_6 = \frac{28(1 - (\frac{1}{2})^6)}{1 - 1/2}$
$= 55.125$

14. In a geometric series with r = 4 and $a_2 = 24$, which term (n) would have the value $a_n = 6291456$?

$a_1 = 6$

$6{,}291{,}456 = 6(4)^{n-1}$
$1048576 = 4^{n-1}$
$\log_4 1048576 = n-1$
$10 = n-1$

$n = 11$

15. A bouncy ball is dropped from the top of Hinsdale Central, 20 feet above the ground. It hits the ground and bounces 62% of its original height on each subsequent bounce.

a. What is the total distance that the ball travels when it hits the ground for the 7th time?

$2 \cdot \sum_{k=1}^{6} 12.4(.62)^{k-1} + 20 \approx 81.56$ feet

b. What is the total distance that the ball travels before it stops?

$2 \sum_{k=1}^{\infty} 12.4(.62)^{k-1} + 20 \approx 85.26$ feet

16. The bob of a pendulum swings through an arc 36 in long on its first swing. If each successive swing is approximately 4/5 of the length of the preceding swing, how far will the bob travel before coming to rest?

$\sum_{k=1}^{\infty} 36\left(4/5\right)^{k-1} = \frac{36}{1 - 4/5} = 180$