

Keys

1. Write a general term (a_n) for the sequence:

a. $\frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \dots$
 $n=1, 2, 3$

$$a_n = \frac{n+2}{n+3}$$

b. $3, -15, 75, -375$

$$a_n = (-1)^{n+1} \cdot 3(5)^{n-1}$$

c. $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \dots$
 $n=1, 2, 3, 4$

$$a_n = \frac{n+1}{n+2}$$

2. Write each series using summation notation with the summing index k starting at $k=1$.

a. $-9 + 16 - 25 + 36$
 $n=1 \quad 2 \quad 3 \quad 4$

$$\sum_{k=1}^4 (-1)^k (k+2)^2$$

b. $\frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6}$

$$\sum_{k=1}^5 \frac{1}{2^{k+1}}$$

c. $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7}$

$$\sum_{k=1}^7 (-1)^{k+1} \frac{1}{k}$$

3. Given the following sequences, write both the EXPLICIT and RECURSIVE formulas.

a. $-12, -4, 4, 12, 20, \dots$

EXP: $a_n = -12 + (n-1)8$

REC: $a_n = a_{n-1} + 8, n \geq 2$
 $a_1 = -12$

b. $\frac{3}{4}, 3, 12, 48, \dots$

EXP: $a_n = \frac{3}{4}(4)^{n-1}$

REC: $a_n = a_{n-1} \cdot 4, n \geq 2$
 $a_1 = 3/4$

c. $2, -10, 50, -250$

EXP: $a_n = 2(-5)^{n-1}$

REC: $a_n = a_{n-1} \cdot -5, n \geq 2$
 $a_1 = 2$

$a = 4 + -5(87-1)$

4. Find the 87th term in an arithmetic sequence with $a_1 = 4$ and $d = -5$.

87

$$a_{87} = -426$$

5. Find the 15th term in a geometric sequence with $a_2 = 7$ and $r = 2$.

$$a_1 = \frac{7}{2}$$

$$a_{15} = \frac{7}{2}(2)$$

$$a_{15} = 57,344$$

6. Find the 13th term of:

a) a geometric sequence with: $a_3 = 3$ and $a_7 = 768$

$$768 = 3 \cdot r^4$$

$$256 = r^4$$

$$\pm 4 = r$$

$$a_1 = \frac{3}{16}$$

$$\text{so } a_{13} = \frac{3}{16}(\pm 4)^{12}$$

$$a_{13} = 3,145,728$$

b) an arithmetic sequence with: $a_5 = -16$ and $a_{11} = 14$

$$d = \frac{14 + 16}{11 - 5} = \frac{30}{6} = 5$$

$$a_1 = -36$$

$$a_{13} = -36 + (13 - 1)5$$

$$a_{13} = -36 + 60$$

$$a_{13} = 24$$

7. We studied arithmetic and geometric series. Which converge and which diverge? Why?

↓
ONLY
 $a_n \neq 0$

↑
 $0 < |r| < 1$

↑
 $|r| > 1$

8. Consider the following geometric series formulas. What is the difference and how do we get from the first to the second and why?

$$S_n = \frac{a_1(1 - r^n)}{1 - r}, \quad S_\infty = \frac{a_1}{1 - r}$$

if $-1 < r < 1$

as $n \rightarrow \infty$

this $(r^n) \rightarrow 0$

9. First determine if the series is arithmetic or geometric, or neither. Then, find the indicated sum.

a. $\sum_{k=1}^8 8\left(\frac{1}{4}\right)^k$ *geometric*
 $S_8 = \frac{2(1 - \frac{1}{4}^8)}{1 - \frac{1}{4}}$

$S_8 \approx 2.66$

b. $\sum_{k=1}^4 (-1)^{k-1} (4k+1)$ *neither*
 $5 - 9 + 13 - 17$
 $\boxed{-8}$

c. $\sum_{k=1}^{25} (7-2k)$ *arithmetic*
 $S_{25} = \frac{25}{2}(5 + -43)$
 $\boxed{S_{25} = -475}$

d. $\sum_{k=1}^{\infty} \left(-\frac{1}{2}\right)^{k+1}$ *infinite geometric*
 $S_{\infty} = \frac{\frac{1}{4}}{1 + \frac{1}{2}} = \frac{1}{4} \cdot \frac{2}{3} = \boxed{\frac{1}{6}}$

10. Find the sum of the positive multiples of 3 from 4 to 99 (inclusive)

6, 9, 12, ..., 99

$a_1 = 6$ $d = 3$
 $a_n = 99$

$99 = 6 + 3(n-1)$
 $n = 32$

arithmetic
 $S_{32} = \frac{32}{2}(6 + 99)$

$\boxed{S_{32} = 1680}$

11. Find the sum of all even integers from 5 to 42.

6, 8, 10, ..., 42

$a_1 = 6$ $d = 2$
 $a_n = 42$

$42 = 6 + (n-1)2$
 $n = 19$

$S_{19} = \frac{19}{2}(6 + 42)$

$\boxed{S_{19} = 456}$

12. Careful here!!! ☺ Find: $\sum_{k=7}^{40} -3 + 2(k-1)$

$\sum_{k=1}^{40} - \sum_{k=1}^6 \Rightarrow \frac{40}{2}(-3 + 75) - \frac{6}{2}(-3 + 7)$
 $1440 - 12$
 1428

13. Determine the summation of the following series:

a. $22 + 18 + 14 + \dots + -26$

$$\begin{aligned} a_1 &= 22 & -26 &= 22 - 4(n-1) \\ d &= -4 & -26 &= 22 - 4n + 4 \\ & & n &= 13 \end{aligned}$$

$$S_{13} = \frac{13}{2}(22 - 26)$$

$$S_{13} = -26$$

b. $28 + 14 + 7 + \dots + 0.875$

$$\begin{aligned} a_1 &= 28 & .875 &= 28\left(\frac{1}{2}\right)^{n-1} \\ r &= \frac{1}{2} & \frac{1}{32} &= \frac{1}{2}^{n-1} \end{aligned}$$

$$S_n = \frac{28(1 - (\frac{1}{2})^n)}{1 - \frac{1}{2}}$$

$$= 55.125$$

$$\left(\frac{1}{2}\right)^5 = \frac{1}{2}^{n-1}$$

$$n = 6$$

14. In a geometric series with $r = 4$ and $a_2 = 24$, which term (n) would have the value $a_n = 6291456$?

$$a_1 = 6 \quad 6,291,456 = 6(4)^{n-1}$$

$$1048576 = 4^{n-1}$$

$$\log_4 1048576 = n-1$$

$$10 = n-1$$

$$n = 11$$

15. A bouncy ball is dropped from the top of Hinsdale Central, 20 feet above the ground. It hits the ground and bounces 62% of its original height on each subsequent bounce.

a. What is the total distance that the ball travels when it hits the ground for the 7th time?

$$2 \cdot \sum_{k=1}^6 12.4(.62)^{k-1} + 20 \approx 81.56 \text{ feet}$$

b. What is the total distance that the ball travels before it stops?

$$2 \sum_{k=1}^{\infty} 12.4(.62)^{k-1} + 20 \approx 85.26 \text{ feet}$$

16. The bob of a pendulum swings through an arc 36 in long on its first swing. If each successive swing is approximately $\frac{4}{5}$ of the length of the preceding swing, how far will the bob travel before coming to rest?

$$\sum_{k=1}^{\infty} 36\left(\frac{4}{5}\right)^{k-1} = \frac{36}{1 - \frac{4}{5}} = 180$$