

Simplify.

$$1. -256^{\frac{3}{4}}$$

$$= -(256^{1/4})^{-3}$$

$$= -\frac{1}{64}$$

$$4. \left( \frac{30a^{-5}b^9}{\frac{3}{5}a^{-7}b^4} \right)^2$$

$$= (50a^2b^5)^2$$

$$= 2500a^4b^{10}$$

$$6. \frac{4}{\sqrt{7}+\sqrt{3}} \cdot \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}}$$

$$= \frac{4\sqrt{7}-4\sqrt{3}}{7-3}$$

$$= \frac{4\sqrt{7}-4\sqrt{3}}{4}$$

$$= \sqrt{7}-\sqrt{3}$$

$$11. \log_{3/2} \frac{16}{81}$$

$$\left(\frac{3}{2}\right)^x = \frac{16}{81}$$

$$\left(\frac{2}{3}\right)^{-x} = \left(\frac{2}{3}\right)^4$$

$$x = -4$$

$$14. 64^{3 \log_4 2}$$

$$= 64^{\log_4 8}$$

$$= (4^3)^{\log_4 8}$$

$$= 4^{\log_4 8^3} = 8^3 = 512$$

OR  $64^{3/2} = 8^3$

$$2. 3x^3 \sqrt[3]{81x^4y^{12}z^{25}}$$

$$= 3x^3 \sqrt[3]{27 \cdot 3 \cdot x^3 \cdot x \cdot y^{12} \cdot z^{24} \cdot z}$$

$$= 3x \cdot 3 \cdot x \cdot y^4 z^8 \sqrt[3]{3xz}$$

$$= 9x^2y^4z^8 \sqrt[3]{3xz}$$

$$4. \frac{6}{\sqrt[5]{128x^4} \cdot 2^7 \cdot x^4}$$

$$= \frac{6}{2^5 \sqrt[5]{2^2 \cdot x^4}}$$

$$= \frac{3}{\sqrt[5]{2^2 \cdot x^4}} \cdot \frac{\sqrt[5]{2^2 \cdot x}}{\sqrt[5]{2^2 \cdot x}}$$

$$= \frac{3 \sqrt[5]{8x}}{2x}$$

$$9. \log_{1/2} 16$$

$$\left(\frac{1}{2}\right)^x = 16$$

$$2^{-x} = 2^4$$

$$x = -4$$

$$12. \log_8 (\log_4 (\log_{100}))$$

$$= \log_8 (\log_4 2)$$

$$= \log_8 \frac{1}{2}$$

$$= -\frac{1}{3}$$

$8^x = \frac{1}{2}$   
 $2^{3x} = 2^{-1}$

$$15. e^{\ln 8}$$

$$= 8$$

$$3. y(3x^3y^5z)^2 (x^8y^{-1}z^{-3})^{-2}$$

$$y \cdot 9x^6y^{10}z^2 \cdot x^{-16}y^2z^6$$

$$= \frac{9x^6y^{11}z^2 \cdot y^2 \cdot z^6}{x^{16}}$$

$$= \frac{9y^{13}z^8}{x^{10}}$$

$$5. 3\sqrt{28} + \frac{21}{\sqrt{7}} - \sqrt[5]{343}$$

$$= 3 \cdot 2\sqrt{7} + \frac{21\sqrt{7}}{7} - (7^3)^{1/5}$$

$$= 6\sqrt{7} + 3\sqrt{7} - \sqrt{7}$$

$$= 8\sqrt{7}$$

$$10. \left(-\frac{1}{32}\right)^{-2/5}$$

$$\left(-\frac{1}{2}\right)^{-2}$$

$$= 4$$

$$13. (\log_7 9)(\log_3 7)$$

$$\frac{\log 9}{\log 7} \cdot \frac{\log 7}{\log 3}$$

$$= \frac{\log 9}{\log 3} = \log_3 9 = 2$$

$$16. \ln e^{2^3}$$

$$= 8$$

17) Use  $\log 2 = a$ ,  $\log 3 = b$  and  $\log 7 = c$ .

a)  $\log 42$

$$\log(2 \cdot 3 \cdot 7) = A + B + C$$

b)  $\log 2\sqrt{7}$

$$\begin{aligned} \log(2 \cdot \sqrt{7}) &= \log(2 \cdot 7^{1/2}) \\ &= A + \frac{1}{2}C \end{aligned}$$

c)  $\log 280 - 3\log 49$

$$\begin{aligned} \log\left(\frac{280}{(7^2)^3}\right) &= \log\left(\frac{7 \cdot 4 \cdot 10}{7^6}\right) \\ &= \log\left(\frac{2 \cdot 2 \cdot 10}{7^5}\right) = 2A + 1 - 5C \end{aligned}$$

18) Write each expression as a single logarithm.

a)  $\log_3 12 + \log_3 6$

$$\begin{aligned} \log_3(12 \cdot 6) \\ = \log_3 72 \end{aligned}$$

b)  $\log_2 8 - \log_2 4$

$$\begin{aligned} \log_2\left(\frac{8}{4}\right) \\ = \log_2 2 \\ = 1 \end{aligned}$$

c)  $3\log_5 2 + \log_5 4 - 2\log_5 8$

$$\begin{aligned} \log_5 8 + \log_5 4 - \log_5 64 \\ = \log_5\left(\frac{8 \cdot 4}{64}\right) \\ = \log_5\left(\frac{1}{2}\right) \end{aligned}$$

19. Solve for the variable. Show all work. Give an exact answer and a decimal approximation to 2 decimal places.

a)  $64^x = \frac{1}{32}$

$$\begin{aligned} 2^{6x} &= 2^{-5} \\ x &= \frac{-5}{6} = -0.83 \end{aligned}$$

b)  $27^x = 9^{x-6}$

$$\begin{aligned} 3^{3x} &= (3^2)^{x-6} \\ 3x &= 2x - 12 \\ x &= -12 \end{aligned}$$

c)  $1.06^a = 4.1$

$$\begin{aligned} \log_{1.06} 4.1 &= a \\ a &= 24.22 \end{aligned}$$

d)  $3\log(x-3) + 4 = 5$

$$\begin{aligned} 3\log(x-3) &= 1 \\ \log(x-3) &= \frac{1}{3} \\ 10^{1/3} &= x-3 \end{aligned}$$

$$\begin{aligned} x &= 3 + 10^{1/3} \\ &= 5.15 \end{aligned}$$

e)  $5 + \frac{2}{3}(10)^{3-x} = 250$

$$\begin{aligned} \frac{2}{3}10^{3-x} &= 245 \\ 10^{3-x} &= 367.5 \\ \log 367.5 &= 3-x \end{aligned}$$

$$\begin{aligned} x &= 3 - \log 367.5 \\ &= 0.43 \end{aligned}$$

f)  $\log_3(a^4) = 2$

$$\begin{aligned} 3^2 &= a^4 \\ \pm (3^2)^{1/4} &= a \\ 3^{1/2} &= a = \pm 1.73 \end{aligned}$$

g)  $x + 5 = \log_{1/3} 100$

$$\begin{aligned} x &= \log_{1/3} 100 - 5 \\ &= -9.19 \end{aligned}$$

h)  $\ln(x) - 6 = -\ln(x+1)$

$$\begin{aligned} \ln x + \ln(x+1) &= 6 \\ \ln(x^2+x) &= 6 \\ e^6 &= x^2+x \\ 0 &= x^2+x-e^6 \end{aligned}$$

OR  $y_1 = 42$

$$x = \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot (-e^6)}}{2}$$

$\rightarrow 19.59, -20.59$

i)  $7^{3x-1} = 11^{x+2}$

$$\begin{aligned} (3x-1)\log 7 &= (x+2)\log 11 \\ 3Ax - A &= Bx + 2B \end{aligned}$$

$$\begin{aligned} A &= \log 7 \\ B &= \log 11 \\ 3Ax - Bx &= 2B + A \\ x &= (2B + A) / (3A - B) = 1.96 \end{aligned}$$

20. State the domain and range of  $y = 2 - \log_b(x-4)$

$$= -\log_b(x-4) + 2$$

$$D: (4, \infty)$$

$$R: (-\infty, \infty)$$

21. Describe the transformations from the parent graph:  $f(x) = 3^x$

a.  $g(x) = \frac{1}{2}(3)^{x-7} + 4$

① shrink vert baf $\frac{1}{2}$

② up 4

Right 7

b.  $g(x) = -(3)^{2x+3} - 9$

① reflect across x-axis

② down 9

① left + 3  
② shrink hor.  
baf $\frac{1}{2}$

22. Given the function:  $f(x) = \log_2 x$

a. What is the equation of vertical asymptote?

$$x = 0$$

b. You arrive at  $g(x)$  by reflecting  $f(x)$  over the x-axis, shifting it up 2, right 3, then shrinking vertically by a factor of  $\frac{1}{3}$  (in this given order), what is the equation of  $g(x)$ ?

$$g(x) = \frac{1}{3} \cdot [-\log_2(x-3) + 2]$$

c. What is the equation of the new vertical asymptote?

$$x = 3$$

23) Hinsdale Central purchased a new copy machine for \$7,500. It is expected to depreciate at a rate of 13% per year.

a) What will be the value of the copy machine in 2 years?

$$A = 7,500(1 - .13)^2$$
$$= \$5,676.75$$

b) When will the copy machine be valued at \$1000?

$$1000 = 7,500(1 - .13)^x$$

$$.13 = .87^x$$

$$\log .87 \cdot 13 = x$$

$$x = 14.47 \text{ years}$$

24. The half life of a given substance is 7 months. If you start out with 28mg, how much will you have after:

a. 5 months

$$A = 28(.5)^{5/7} = 17.07 \text{ mg}$$

b. 1 year

$$A = 28(.5)^{12/7} = 8.53 \text{ mg}$$

c. How long does it take to deteriorate down to only 1 mg?

$$1 = 28(.5)^{t/7}$$

$$.04 = (.5)^{t/7}$$

$$\log .5 \cdot .04 = t/7$$

$$t = 33.65 \text{ months}$$

25. The value of your home has only been appreciating in value by about 1.2% per year since you bought it in 1998. The current value in 2014 of your home is \$329,000. How much did you purchase it for?

$$329,000 = P(1 + .012)^{16}$$

$$P = \$271,836.46$$

$$\text{OR } A = 329000(1 + .012)^{-16}$$

$$= \$271,836.46$$

26. You put \$15,000 in a bank CD compounding continuously. You have \$17,045 after 15 years. At what rate did you put the money in the CD?

$$17045 = 15000e^{r \cdot 15}$$

$$1.14 = e^{15r}$$

$$\ln 1.14 = 15r$$

$$r = .0085$$

$$\boxed{.85\%}$$

27. During the first stages of an epidemic, the number of sick people increases exponentially with time. Suppose that at the start of the epidemic, there are 40 people sick. By the third day, 200 people are sick.

a. Define the variables and ordered pairs.

$t = \text{time in days}$  (0, 40)  
 $P = \# \text{ of people sick}$  (3, 200)

b) Write an equation that describes the situation.

$$\begin{array}{r} 40 \\ 0 \\ \hline 1 \\ 2 \\ 3 \\ 200 \end{array}$$

$$40b^3 = 200$$

$$b^3 = 5$$

$$b = 1.71$$

$$P = 40(1.71)^t$$

c) How many people will be sick on the sixth day?

$$P = 40(1.71)^6$$

d) When will the number of sick people reach 7000?

$$7000 = 40(1.71)^t$$

$$175 = 1.71^t$$

$$\log_{1.71} 175 = t$$

$$t = 9.63 \text{ days}$$