

Algebra 2 Trig H
R4, 4.4 Review

Name: _____

Graph the following rational functions.

1. $f(x) = \frac{2x+2}{x^2-1} = \frac{2(x+1)}{(x+1)(x-1)} = \frac{2}{x-1}$

V.A.: $x=1$

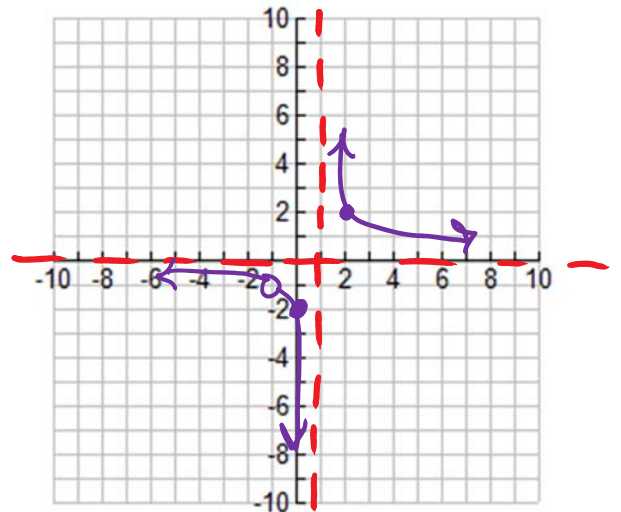
H.A.: $y=0$

x-intercepts: None

y-intercept: $(0, -2)$

Hole: $(-1, -1)$

Extra point: $(2, 2)$



2. $h(x) = \frac{x^2-3x}{x^2+x-12} = \frac{x(x-3)}{(x+4)(x-3)} = \frac{x}{x+4}$

V.A.: $x=-4$

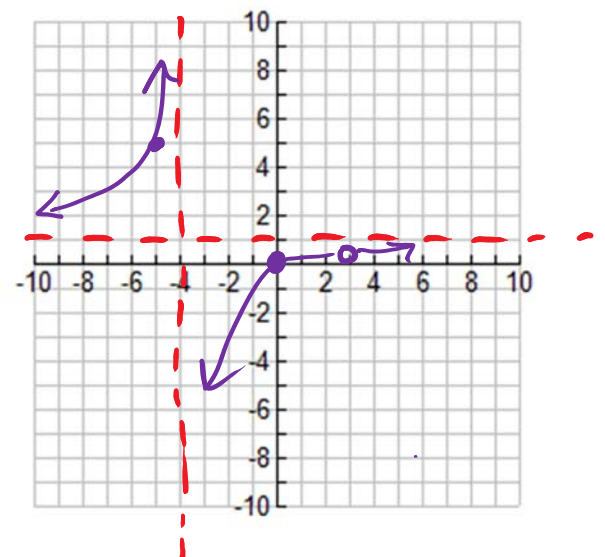
H.A.: $y=1$

x-intercepts: $(0, 0)$

y-intercept: $(0, 0)$

Hole: $(3, 3/7)$

Extra point: $(-5, 5)$



3. $h(x) = \frac{5-x}{x^2+5}$

V.A.: None

H.A.: $y=0$

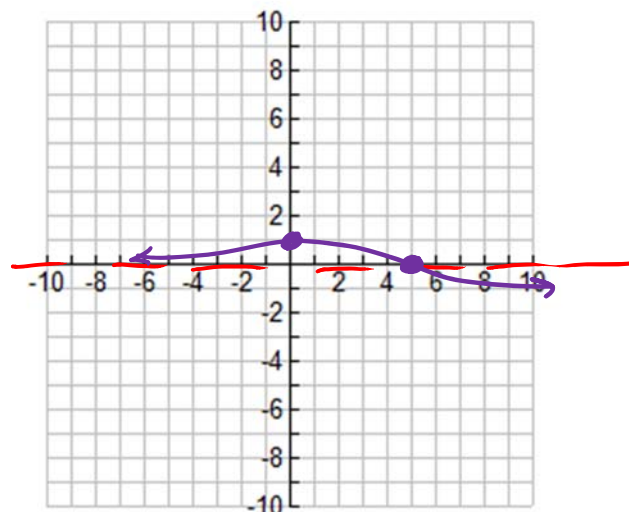
x-intercepts: $(5, 0)$

y-intercept: $(0, 1)$

Hole: None

Extra point: $(10, -1/21)$
 $(-5, 1/3)$

An example of crossing the NA!



4. Write a rational function that has a vertical asymptote at $x=3$, a horizontal asymptote at $y=0$, and a removable discontinuity (hole) at $(-1, -1/4)$.

$$f(x) = \frac{(x+1)}{(x-3)(x+1)}$$

← larger degree
⇒ HA $y=0$

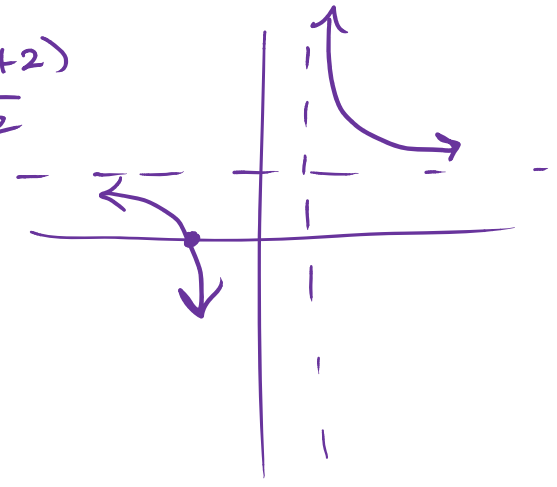
5. Write a rational function that has no horizontal asymptote but does have a removable discontinuity (hole) at $(-2, -1)$.

$$f(x) = \frac{(x+2)(x+1)}{(x+2)}$$

← larger degree
⇒ HA doesn't exist

6. Evaluate the following limits for $m(x) = \frac{2x+4}{x-2}$.

$$= \frac{2(x+2)}{x-2}$$



$$\begin{aligned} \lim_{x \rightarrow \infty} m(x) &= 2 \\ \lim_{x \rightarrow -\infty} m(x) &= 2 \\ \lim_{x \rightarrow 2^-} m(x) &= -\infty \\ \lim_{x \rightarrow 2^+} m(x) &= \infty \end{aligned}$$

Simplify the following.

7. $\frac{-6}{7} \cdot \frac{21}{5} \div \left(\frac{9}{50} \cdot \frac{18}{20} \right)$

$$= \frac{-6 \cdot 21 \cdot 50 \cdot 20}{7 \cdot 5 \cdot 9 \cdot 9}$$

$$= -\frac{40}{9}$$

8. $\frac{x^2+4x-5}{x^2+x-2} \cdot \frac{x^2-4x-12}{x^2+x-20}$

$$= \frac{(x+5)(x-1)}{(x+2)(x-1)} \cdot \frac{(x-6)(x+2)}{(x+5)(x-4)}$$

$$= \frac{x-6}{x-4}$$

9. $\frac{1 + \frac{1}{x}}{x - \frac{1}{x}}$

$$\frac{x+1}{x^2-1} = \frac{x+1}{(x+1)(x-1)} = \frac{1}{x-1}$$

10. $\frac{7}{xy} - \frac{5y}{x} + \frac{3}{y}$

$$\frac{7 - 5y^2 + 3x}{xy}$$

$$11. \left(\frac{3}{x-2} - \frac{1}{x+1} \right) \div \frac{x+4}{x-2}$$

$$\frac{[3(x+1) - (x-2)] \cdot \cancel{x-2}}{(x+1)\cancel{(x-2)}} \cdot \frac{x+4}{x+4}$$

$$= \frac{2x+5}{(x+1)(x+4)}$$

$$12. \frac{2x^2+5x-3}{3x^2+2x-21} \div \frac{2x^2+9x-5}{x^2-1}$$

$$\frac{\cancel{(2x-1)}(x+3)}{(3x-7)\cancel{(x+3)}} \cdot \frac{(x-1)(x+1)}{\cancel{(2x-1)}(x+5)}$$

$$= \frac{(x-1)(x+1)}{(3x-7)(x+5)}$$

$$13. \frac{7}{3x-6} - \frac{3x}{2-x}$$

$$\frac{7}{3(x-2)} - \frac{3x}{-1(x-2)}$$

$$= \frac{7}{3(x-2)} + \frac{9x}{3(x-2)}$$

$$= \frac{9x+7}{3(x-2)}$$

$$14. \frac{x^2}{3x-1} - \frac{x+4}{x-4} + \frac{2x+5}{x+1}$$

$$\frac{x^2(x-4)(x+1) - (x+4)(3x-1)(x+1) + (2x+5)(3x-1)(x-4)}{\text{denom}}$$

$$= \frac{x^2(x^2-3x-4) - (x+4)(3x^2+2x-1) + (2x+5)(3x^2-13x+4)}{\text{denom}}$$

$$= \frac{x^4 - 3x^3 - 4x^2 - 3x^3 - 2x^2 + x - 12x^2 - 8x + 4 + 6x^3 - 26x^2 + 8x + 15x^2 - 65x + 20}{\text{denom}}$$

$$= \frac{x^4 - 29x^2 - 64x + 24}{(3x-1)(x-4)(x+1)}$$

Solve for x.

$$15. \frac{3}{x+1} + \frac{2}{x-4} = \frac{4x-11}{x^2-3x-4}$$

$$3(x-4) + 2(x+1) = 4x-11$$

$$3x-12+2x+2=4x-11$$

$$5x-10=4x-11$$

$$x = -1$$

$$16. 3 - \frac{22}{x+5} = \frac{6x-1}{2x+7}$$

$$3(2x+7)(x+5) - 22(2x+7) = (6x-1)(x+5)$$

$$3(2x^2+17x+35) - 44x - 154 = 6x^2+29x-5$$

$$= 6x^2+29x-5$$

$$\cancel{6x^2} + 51x + 105 - 44x - 154 = \cancel{6x^2} + 29x - 5$$

$$-44 = 22x$$

$$-2 = x \quad \checkmark$$

$$17. \frac{x}{x^2-2x+1} = \frac{2}{x+1} + \frac{4}{x^2-1}$$

$$(x-1)(x-1) \quad (x-1)(x+1)$$

$$x(x+1) = 2(x-1)(x-1) + 4(x-1)$$

$$x^2+x = 2x^2 - 4x + 2 + 4x - 4$$

$$0 = x^2 - x - 2$$

$$0 = (x-2)(x+1)$$

$$x = 2, -1$$

$$18. \frac{1}{x-3} - \frac{1}{x^3-5x^2+6x} = \frac{x-7}{x^2-2x}$$

$$x(x-3)(x-2) \quad x(x-2)$$

$$x(x-2) - 1 = (x-3)(x-7)$$

$$x^2 - 2x - 1 = x^2 - 10x + 21$$

$$8x = 22$$

$$x = \frac{22}{8} = \frac{11}{4} \quad \checkmark$$

$$19. \frac{1}{x^2-3x} + \frac{x+2}{3-x} = 1$$

$$1 - x(x+2) = x(x-3)$$

$$1 - x^2 - 2x = x^2 - 3x$$

$$0 = 2x^2 - x - 1$$

$$0 = (2x+1)(x-1)$$

$$x = -\frac{1}{2}, 1 \quad \checkmark$$

$$20. \frac{1}{4} + \frac{1}{2x^2-16x+24} = \frac{1-x}{24-4x} \rightarrow \frac{x-1}{4(6-x)}$$

$$\frac{2(x^2-8x+12)}{2(x-6)(x-2)} \quad 4(6-x)$$

$$(x-6)(x-2) + 2 = (x-1)(x-2)$$

$$x^2 - 8x + 12 + 2 = x^2 - 3x + 2$$

$$12 = 5x$$

$$12/5 = x \quad \checkmark$$