Trig Honors
Conditional Probability

1. Americans recycle more and more material through municipal waste collection each year. Use the information in the table, based on a recent year.

Millions of tons of municipal waste collected in the United States in one year			
Material	**Recycled**	**Not recycled**	**Totals**
Paper	34.9	48.9	83.8
Metal	6.5	10.1	16.6
Glass	2.9	9.1	12
Plastic	1.1	20.4	21.5
Other	15.3	67.8	83.1
Totals	60.7	156.3	217

what does this total yield?

a. Find the totals.

b. Find the probability that a sample of recycled waste was paper. $\frac{34.9}{60.7} = 0.575$

c. Find the probability that a sample of not recycled waste was paper. $\frac{48.9}{156.3} = 0.313$

d. Find the probability that a sample of paper was not recycled. $\frac{48.9}{83.8} = 0.584$

why 1 here?

e. Find the probability that a sample of paper was recycled. $\frac{34.9}{83.8} = 0.416$

Now you try!

f. Find the probability that a sample of recycled waste was plastic. $\frac{1.1}{60.7} = 0.018$

g. Find the probability that a sample of not recycled waste was plastic. $\frac{20.4}{156.3} = 0.131$

h. Find the probability that a sample of plastic was not recycled. $\frac{20.4}{21.5} = 0.949$

i. Find the probability that sample of plastic was recycled. $\frac{1.1}{21.5} = 0.051$

j. Find the probability that a sample was recycled. $\frac{60.7}{217} = 0.280$

k. Find the probability that a sample was plastic. $\frac{21.5}{217} = 0.099$

2. Researchers asked shampoo users whether they apply shampoo directly to the head, or indirectly using a hand.

	Directly onto head	Into hand first	
Male	2	18	
Female	6	24	20
	8	42	~~30~~ 50

a. How many people were surveyed? 50

b. Find the probability that the person was male. $\frac{20}{50} = \frac{2}{5} = 0.4$

c. Find the probability that the person was female. $\frac{30}{50} = \frac{3}{5} = 0.6$

d. Find the probability that the person put shampoo directly on their head. $\frac{8}{50} = \frac{4}{25} = 0.16$

e. Find the probability that a male puts shampoo into his hand first. $\frac{18}{20} = \frac{9}{10} = 0.9$

f. Find the probability that a person that puts shampoo directly on the head is female. $\frac{6}{8} = \frac{3}{4} = 0.75$

3. Conduct a survey in your class. Which shoe do you put on first?

		Dominant hand	
		Right hand	Left hand
First shoe to put on	Right shoe		
	Left shoe		
	First one grabbed		
	Don’t know		

a. P(left handed | left shoe)

b. P(left handed | not the right shoe)

c. P(left shoe | right handed)

d. P(first shoe picked up | right handed)

e. P(don’t know)

4. Use the table to find each probability.

Degree	Male	Female	
Associate's	224	387	611
Bachelor's	547	776	1323
Master's	245	322	567
	1016	1485	2501

a. What is the probability the recipient is male? $\frac{1016}{2501} = 0.406$

b. What is the probability the person earns a bachelor's degree? $\frac{1323}{2501} = 0.529$

c. What is the probability a person earning a master's degree is female? $\frac{322}{567} = 0.568$

d. What is the probability of a female earning a master's degree? $\frac{322}{2501} = 0.129$
(Not conditional)

e. What is the probability that a male recipient earns a bachelor's degree? $\frac{547}{1016} = 0.538$

Summary:

$P(\text{paper} \mid \text{recycled}) = \dfrac{P(\text{recycled paper})}{P(\text{recycled})}$

$P(A \mid B) = \dfrac{P(A \text{ and } B)}{P(B)}$

$P(\text{female} \mid \text{master's}) = \dfrac{P(\text{female master's})}{P(\text{master's})}$

5. Eighty percent of an airline's flights depart on schedule. Seventy-two percent of its flights depart and arrive on schedule. Find the probability that a flight that departs on time also arrives on time.

$$P(\text{arrives OT} \mid \text{departs OT}) = \frac{P(\text{arrives and departs OT})}{P(\text{departs OT})}$$

$$= \frac{0.72}{0.8} = 0.9$$

Homework

1. The table contains characteristics of job applicants. Use the table to find each probability.

		Has experience	
		Yes	No
Has a high school diploma	Yes	54	27
	No	5	4
		59	31

81
9
90

   a. P(has diploma) $\frac{81}{90} = 0.90$

   b. P(has diploma and experience) $= \frac{54}{90} = 0.60$

   c. P(has experience | diploma) $= \frac{54}{81} = 0.667$

   d. P(has no diploma | has experience) $= \frac{5}{59} = 0.085$

2. Suppose A and B are independent events, with P(A) = 0.60 and P(B) = 0.25. Find each probability.

   a. P(A and B) $0.60 \cdot 0.25 = 0.15$

   b. P(A | B) $= \frac{0.15}{0.25} = 0.60$

   c. What do you notice about P(A) and P(A | B)?

   They are equal!

   d. One way to describe A and B as independent events is "The occurrence of B has no effect on the probability of A." Explain how the answer to part c illustrates this relationship.

3. Use the survey results.

- 39% have a pet now and have had a pet.
- 61% do not have a pet now.
- 86% have had a pet.
- 14% do not have a pet now and have never had a pet.

a. Find the probability that a respondent has a pet, given that the respondent has had a pet.

$$P(\text{Has pet} \mid \text{has had pet}) = \frac{P(\text{pet now + before})}{P(\text{before})} = \frac{0.39}{0.86} = 0.453$$

b. Find the probability that a respondent has never had a pet, given that the respondent does not have a pet now.

$$P(\text{never pet} \mid \text{no pet now}) = \frac{P(\text{NO + NO})}{P(\text{NO pet Now})} = \frac{0.14}{0.61} = 0.230$$

4. Suppose that a child has a probability of 0.12 of catching measles and a probability of 0.2 of catching chicken pox in any one given year.

a. If these events are independent of each other, what is the probability that he or she will get both diseases in a given year?

$$0.12 \cdot 0.2 = 0.024$$

b. Suppose that statistics show the following probabilities for getting both diseases in the same year:

P(measles, then chicken pox) = 0.006

P(chicken pox, then measles) = 0.18

Calculate the probability of getting:

i. Chicken pox after measles

$$P(C \mid M) = .006/.12 = 0.05$$

ii. Measles after chicken pox

$$P(M \mid C) = 0.18/0.2 = 0.90$$

c. Based on your answer to part b, what could you conclude about the effects of the two diseases on each other?

we would expect the conditional probabilities to be small if the diseases were truly independent, but P(M|C) is large.

The diseases are NOT independent of each other. Measles makes a person more susceptible to chicken pox.