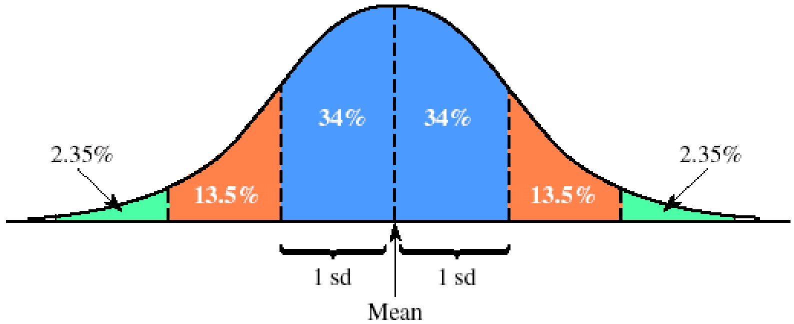
Day 3 Notes – Standard Normal Distribution

Recall:

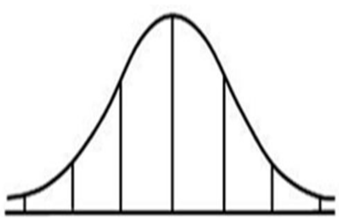


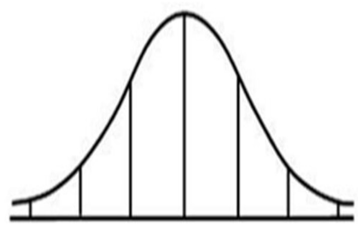
Let's revisit the last problem from the homework: 

The wing lengths of houseflies are normally distributed with a mean of 4.6 millimeters and a standard deviation of 0.4 millimeters.

a. What percent of flies have wing lengths between 4.6 and 5.4? 34% + 13.5% = 47.5%

b. What percent of flies have wing lengths less than 4.1? Now what?

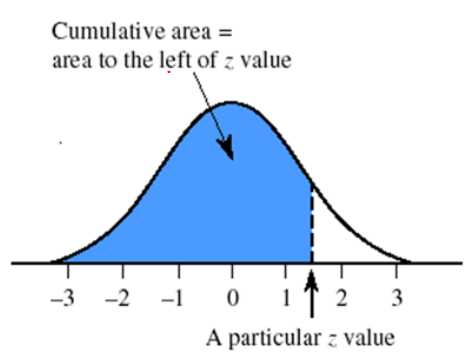




How to find a z score:

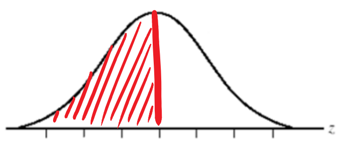
$$z = \frac{x - \mu}{\sigma}$$

The z-score gives us the cumulative area that is to the left of the z-value.



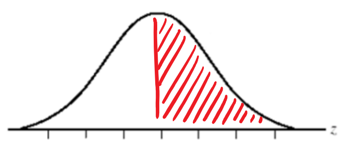
Let's practice using the z-table. Find the following probabilities.

a. $P(z < 0)$



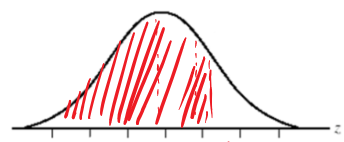
$\boxed{.50}$

b. $P(z > 0)$



$\boxed{.50}$

c. $P(z < 1.42)$

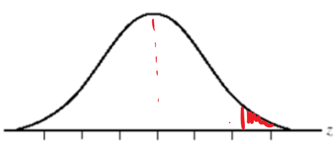


$\boxed{.9222}$ =1.4

d. $P(z < -2.34)$

$\boxed{.0096}$

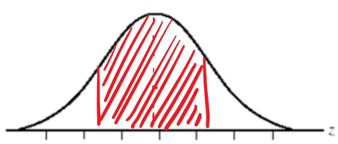
e. $P(z > 2.60)$



$1 - P(z < 2.60)$

$1 - .9953 = \boxed{.0047}$

d. $P(-1.56 < z < 1.3)$



$P(z < 1.3) - P(z < -1.56)$

$.9032 - .0594$

$\boxed{.8438}$

e. $P(|z| < 2.2)$

$-2.2 <$ < 2.2

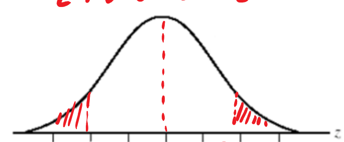
$P(z < 2.2) - P(z < -2.2)$

$.9861 - .0139$ ← discuss comp. (w/ z table)

$\boxed{.9722}$

f. $P(|z| > 2.2)$

$z > 2.2$ or $z < -2.2$



$2(.0139)$

$1 - 0.9722$

$= \boxed{0.0278}$

Ex. 1. A study finds that the weights of infants at birth are normally distributed with a mean of 3270 grams and a standard deviation of 600 grams. An infant is randomly chosen.

a. What is the probability that the infant weighs less than 4170 grams?

$$z = \frac{4170 - 3270}{600} = 1.5 \qquad P(z < 1.5) = 0.9332$$

b. What is the probability that the infant weighs 3990 grams or more?

$$z = \frac{3990 - 3270}{600} = 1.2 \qquad P(z > 1.2) = 1 - 0.8849$$

$$= 0.1151$$

c. What is the probability that infant weighs between 3000 and 3576 grams?

$$z_1 = \frac{3000 - 3270}{600} = -0.45 \qquad z_2 = \frac{3576 - 3270}{600} = 0.51$$

$$P(-.045 < z < 0.51) = 0.6950 - 0.3264$$

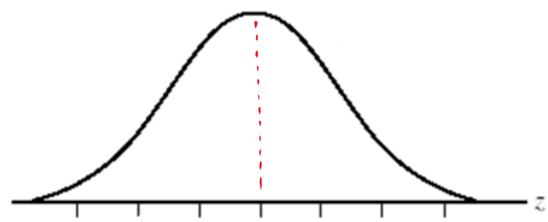
$$= 0.3686$$

Ex. 2 Scientists conducted aerial surveys of a seal sanctuary and recorded the number of seals they observed during each survey. The number of seals they observed were normally distributed with a mean of 73 seals and a standard deviation of 14.1 seals.

a. Find the probability that at most 50 seals were observed during a randomly chosen survey.

$z = \frac{50-73}{14.1} \approx -1.63$

$P(z < -1.63) = \boxed{.0516}$



b. Find the number of seals observed if the z-score is -1.5.

$-1.5 = \frac{x-73}{14.1}$

$-21.15 = x - 73$

$x = 51.85$

Approximately 52 Seals

Look for .8599 in z-table

c. The percent to the left of a z-score is 0.8599. What is the number of seals to make this happen?

So $z = 1.08$

$1.08 = \frac{x-73}{14.1}$

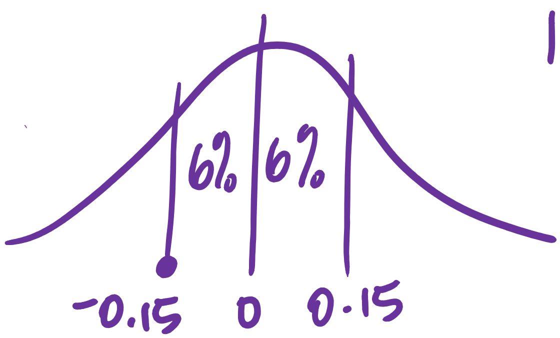
$15.228 = x - 73$

$88.228 = x$

Approximately 88 seals

d. Find the number of seals observed within 6% of the mean.

look @ percentages for 0.44 + 0.56



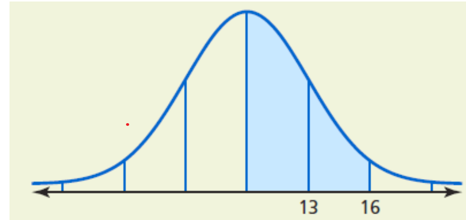
$-0.15 = \frac{x-73}{14.1}$

$0.15 = \frac{x-73}{14.1}$

70.885, 75.115

Day 3 Homework

1. In the figure, the shaded region represents 47.5% of the area under a normal curve with the vertical lines indicating a standard deviation. What are the mean and standard deviation of the normal distribution?

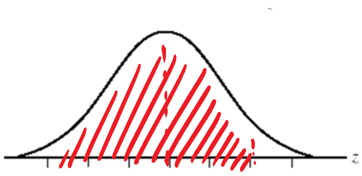


10

$\boxed{\mu=10 \quad \sigma=3}$

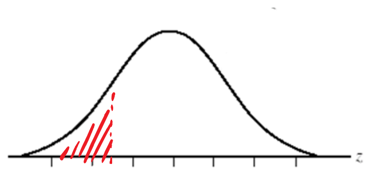
2. Use the z-table to help you answer the questions below. Find the following probabilities:

a. P(z < 2.4)



$\boxed{.9918}$

b. P(z < -1.51)



$\boxed{.0655}$

c. (-1.23 < z < 1.5)

-1.23 1.5

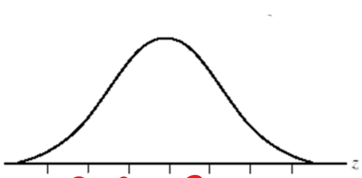
P(z < 1.5) − P(z < -1.23)

.9332 − .1093

$\boxed{.8239}$

d. P(|z| < 1.7)

-1.7 < z < 1.7



0.9109

e. P(|z| > 1.7)

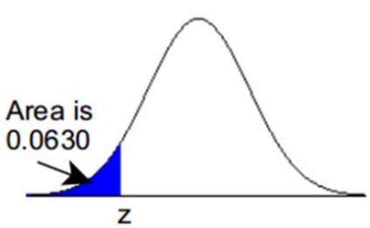
z > 1.7 z < -1.7

1 − 0.9109 = 0.0891

3. Find the value of z from the standard normal distribution that satisfies each of the following conditions.

(Use the value that comes the closest!)

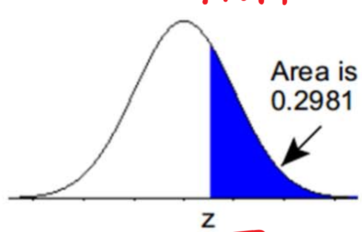
a.



$\boxed{z = -1.53}$

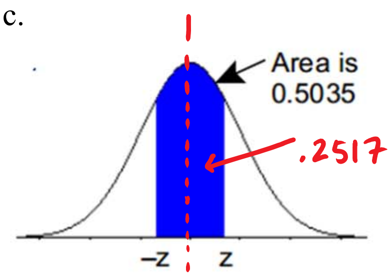
b.

1 − 0.2981
= .7019



$\boxed{z = 0.53}$

c.



.2517

.5 + .2517 = .7517

$\boxed{z = 0.68}$

4. A busy time to visit a bank is during its Friday evening rush hours. For these hours, the waiting times at the drive through window are normally distributed with a mean of 8 minutes and a standard deviation of 2 minutes. You have no more than 11 minutes to do your banking and still make it to your meeting on time. What is the probability that you will be late for the meeting?

$z = \frac{11-8}{2} = 1.5$

$1 - P(z<1.5)$

$1 - .9332$

$\boxed{.0668}$ or 6.8% chance you'll be late

5. The guayule plant, which grows in southwestern United States and in Mexico, is one of several plants that can be used as a source of rubber. In a large group of guayule plants, the heights of the plants are normally distributed with a mean of 12 inches and a standard deviation of 2 inches.

a. What percent of the plants are at most 13 inches? $z = \frac{13-12}{2} = .5$ $P(z<.5) = .6915$ $\boxed{69.15\%}$

b. What percent of the plants are taller than 16 inches? $z = \frac{16-12}{2} = 2$ $1 - P(z<2) =$ $1 - .9772 = .0228$ $\boxed{2.28\%}$

c. What percent of the plants are between 13 inches and 16 inches? $P(z<2) - P(z<.5)$

$.9772 - .6915$

$.2857$ $\boxed{28.57\%}$

6. Elephants have the longest pregnancy of all mammals. One species of elephant has a mean gestation period of 525 days and a standard deviation of 32 days. Their pregnancy length follows an approximately normal distribution.

a. What percent of elephant pregnancies last less than 461 days?

$z = \frac{461-525}{32} = -2$ $P(z<-2) = \boxed{.0228}$

b. The longest 20.9% of all elephant pregnancies last at least how many days?

find z for 79.1% $.81 = \frac{x-525}{32}$ $= 550.92 \sim \boxed{551 \text{ days}}$

c. The middle 68% of elephant pregnancies last between how many days?

Don't overthink it! (within 1 stand. dev)

$525 + 32 = 557$

$525 - 32 = 493$

$\boxed{\text{Between 493 and 557 days}}$