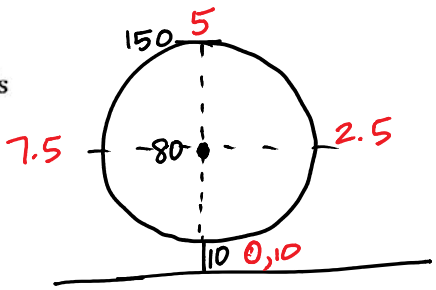
Applications

Wednesday, February 8, 2017 1:40 PM

6.5 Applications
Sinusoidal Functions as Mathematical Models

Navy Pier Ferris Wheel Statistics:

→ Diameter is 140 feet
→ The center is 80 feet off the ground
→ It takes 10 minutes to go around the Ferris wheel one time



Problem #1 The distance a rider on the Ferris wheel is above the ground can be modeled by a sinusoidal graph. Answer the following questions and then plot a graph of the rider's position during one revolution of the wheel.

a. The rider gets on at exactly the lowest point on the wheel. How high above the ground is this point? 10 feet

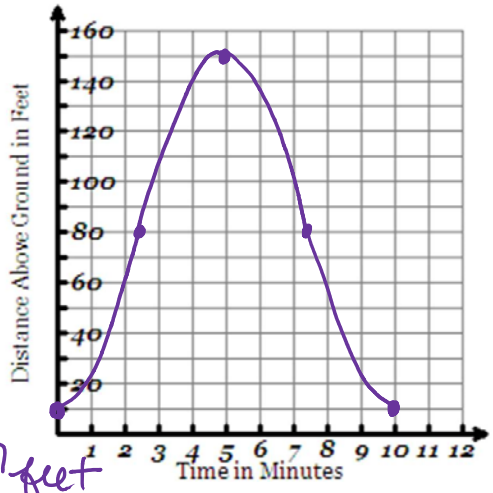
b. If the rider gets on at time zero, when will the rider be back at the lowest point again? 10 min

c. What is the highest point the rider will ever reach above the ground? 150 feet

d. At what time will the rider be at the highest point? 5 min

e. At what times will the rider directly left or right the center of the Ferris wheel? 2.5 min, 7.5 min.

f. Plot these points and sketch a graph of the rider's distance above the ground as a function of the time they've been on the ride. Then write the equation of the sinusoid.



$y = -70\cos\left(\frac{\pi}{5}x\right) + 80$

$\frac{2\pi}{B} = 10$

g. How high above the ground is the rider after 2 minutes?

$-70\cos\left(\frac{\pi}{5} * 2\right) + 80 = 58.37$ feet

h. If the rider stays on for more than one revolution, how high above the ground is he/she after 16 minutes?

136.63 feet

i. Find the first two times the rider is 120 feet above the ground.

$y1$
$y2 = 120$

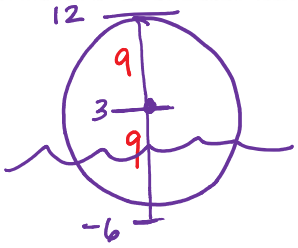
$t = 3.47, 6.53$ min.

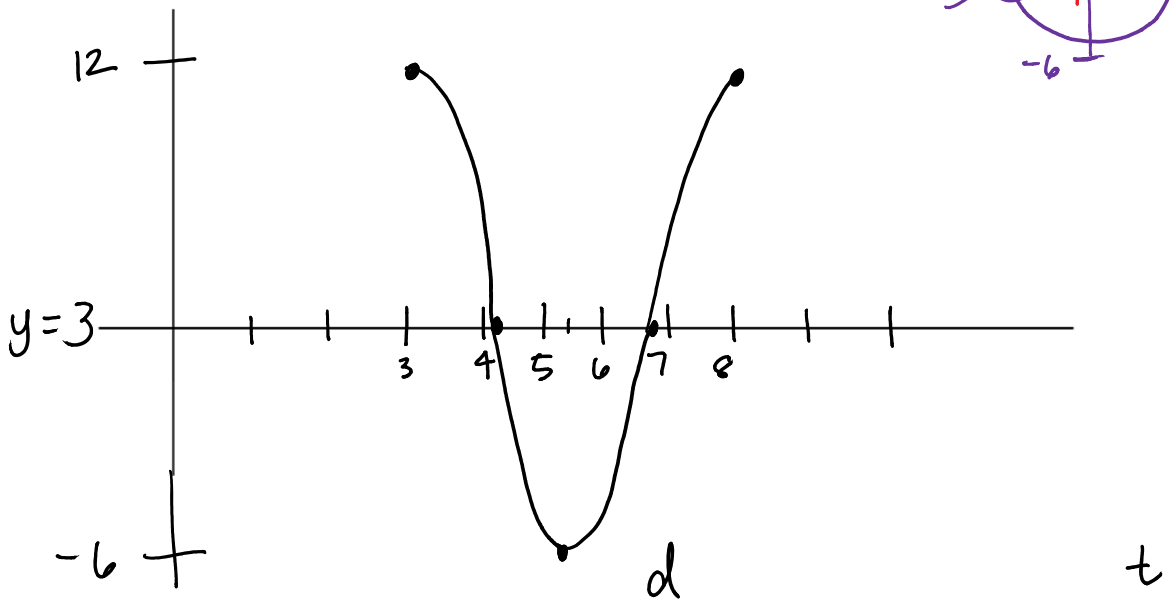
j. Without using your calculator, what are the next two times the rider is 120 feet above the ground? How did you determine these values?

13.47 min, 16.53 min.

PROBLEM #2 Mrs. Green's class takes a field trip to Longford Mill. Andrew decides to get some data while admiring the water wheel. He leans over to get a better look and his hat falls off and gets caught on the outer rim of the wheel. He recognizes that the diameter of the wheel is 18 feet. At time = 3 seconds, his hat is at a maximum point of 12 feet above the surface of the water. The wheel makes one revolution every 5 seconds.

a. Sketch a graph describing the distance *(d)* the hat is above the surface of the water with respect to time, *t*.





b. Write an equation describing the distance the hat is above the surface of the water with respect to time.

$$d = 9\cos\left(\frac{2\pi}{5}(t-3)\right)+3$$

$\frac{2\pi}{B} = 5$

$B = \frac{2\pi}{5}$

c. What is the location of the hat at time t = 5 seconds?

$9\cos\left(\frac{2\pi}{5}\cdot(5-3)\right)+3 = -4.28$

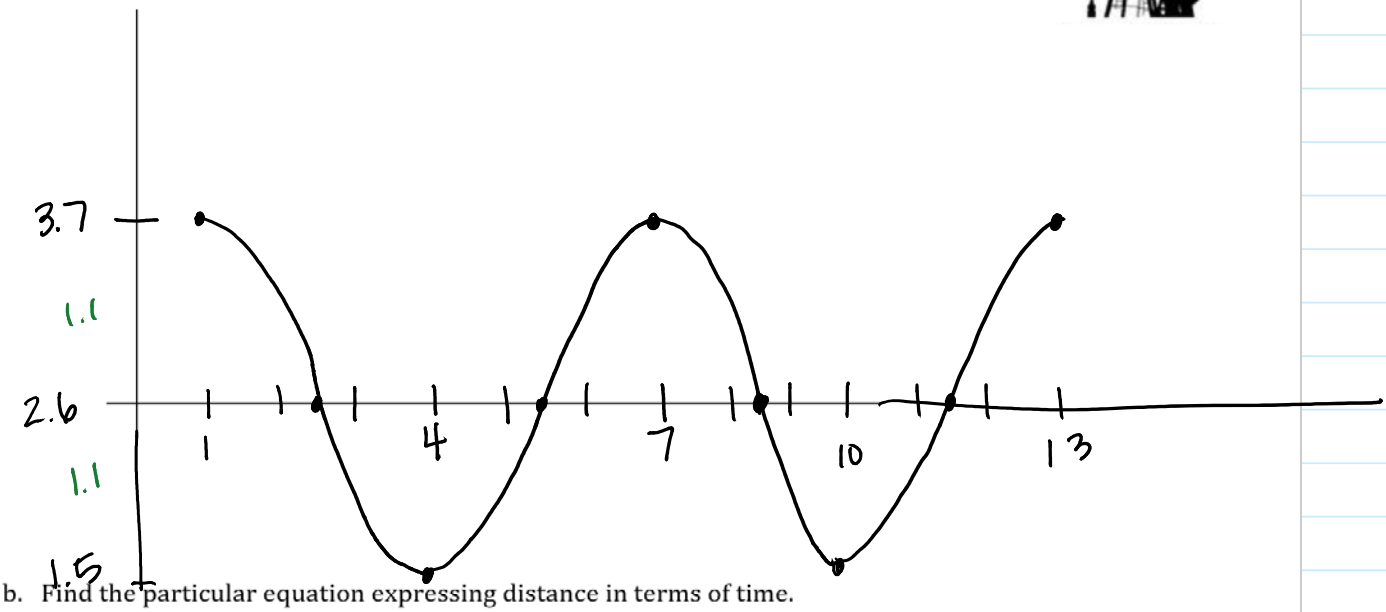
So it is 4.28 feet below the surface

d. Describe all the times the hat will be at a maximum point.

$t = 3 + 5k,\ k \in \mathbb{Z}$

Problem #3 Oil Pump The jack on an oil well goes up and down pumping oil out of the ground. As it does this, the distance the end of the jack is from the ground varies sinusoidally with time. At time t = 1 second, the jack is at its maximum distance from the ground, 3.7 meters. At 4 seconds, the jack is at its minimum distance from the ground, 1.5 meters.

a. Sketch a graph of two cycles of the oil jack going up and down.



b. Find the particular equation expressing distance in terms of time.

$$d = 1.1\cos\left(\frac{\pi}{3}(t-1)\right) + 2.6$$

c. How high above the ground is the jack at 5 seconds? 9.3 seconds?

2.05 feet

1.78 feet

d. Find the first 4 times the jack is at 2 meters above the ground.
(Remember you really only need to do an intersection on your calculator for the first two!)

3.05, 4.95, 9.05, 10.95

HOMEWORK DAY 1:

Problem #4 While hanging out at County Fair, you and your best friend choose to ride the Ferris wheel. Your distance from the ground varies sinusoidally with the time since the wheel began to move. Let t be the number of seconds that have elapsed since the motion of the Ferris wheel began. You find that it takes you 5 seconds to reach the top, 50 feet above the ground and that the wheel makes 1 revolution every 12 seconds. The diameter of the wheel is 45 feet.

a. Sketch a graph of this function.

b. Write the particular equation of this function.

c. What is the lowest height that you reach on the wheel?

d. Predict your height when t = 13 seconds.

Problem #5

The average depth of water at the end of a dock is 6 feet. This varies sinusoidally 2 feet in both directions with the tide. The high tide occurs at 4 AM. At 10 AM you notice the tide is at its lowest.

a. Sketch a graph of this function.

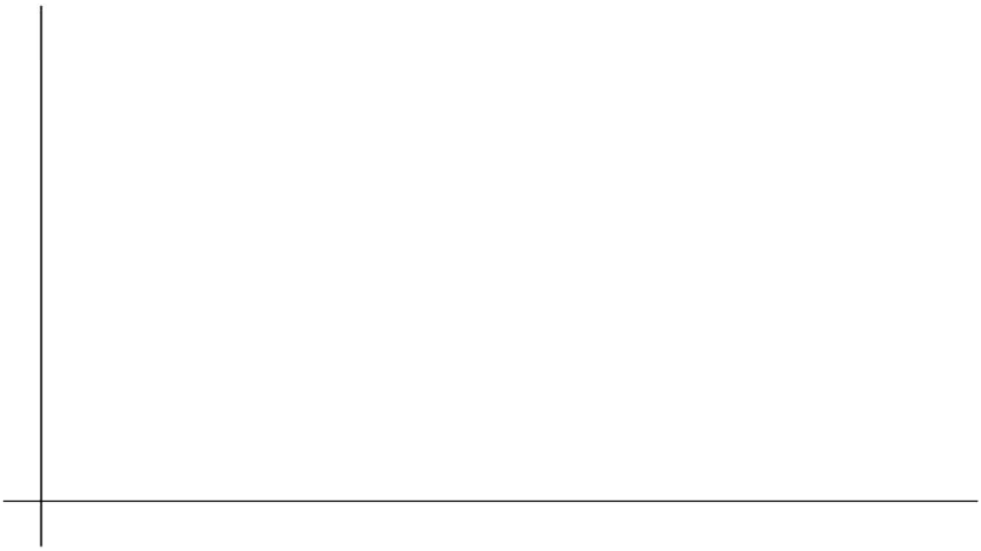
b. Write the particular equation.

c. Where will it be at 3 **PM** (be careful!)?

Problem #6

The population of dragonflies on a small puddle in the San Antonio River varies sinusoidally with time. On October 4th the population was at its maximum of 3200 and on October 14th it was at its minimum of 600

a. Sketch a graph of this function.



b. Write the particular equation.

c. How many dragonflies where there on Oct. 1st?

d. Find the first day in October when there are 1000 dragonflies.