8.1 Proportional Relationships

Consider the proportion $\frac{a}{b}=\frac{c}{d}$ **also known as,** $\frac{1st\ term}{2nd\ term}=\frac{3rd\ term}{4th\ term}$

The 1st and 4th terms are called extremes. The 2nd and 3rd terms are called means.

Mean -Extremes Products Theorem: If $\frac{a}{b}=\frac{c}{d}$, then $ad=bc$.

Means-Extremes Ratio Theorem: If $pq=rs$, then $\frac{p}{r}=\frac{s}{q}, \frac{p}{s}=\frac{r}{q}$, and $\frac{r}{p}=\frac{q}{s}$.

Example 1: $\frac{3}{x}=\frac{7}{14}$, solve for x.

$7x=42$

$x=6$

Example 2: Find the fourth term of a proportion if the 1st three terms are 2, 3, 4.

$\frac{2}{3}=\frac{4}{x}$ $2x=12$ $x=6$

Example 3: If $3x=4y$, find the ratio of x to y.

$\frac{3x}{y}=\frac{4y}{y}$ $\frac{x}{y}=\frac{4}{3}$

Example 4: Find the ratio of x to y. gx – fy = hx + my

$gx-hx=fy+my$

$x(g-h)=y(f+m)$

$\frac{x}{y}=\frac{f+m}{g-h}$

Definition: If the means of a proportion are equal, either mean is called a geometric mean, or mean proportional, between the extremes.

Example 5: Find the geometric mean between 3 and 27.

$\frac{3}{x}=\frac{x}{27}$ $x^2=3\cdot3\cdot3\cdot3$

$x=\pm9$

Example 6: Find the mean proportional between 4 and 16.

$\frac{4}{x}=\frac{x}{16}$ $x^2=4\cdot16$

$x=\pm8$

Other problems:

1. Find the arithmetic mean between 6 and 30.

$\frac{6+30}{2}=18$

2. Find the fourth proportional of 6, 8, and 9.

$\frac{6}{8}=\frac{9}{x}$

$6x=72$

$x=12$

3. Calculate the geometric mean between 16 and 25.

$\frac{16}{x}=\frac{x}{25}$

$x^2=16\cdot 25$

$x=\pm 20$

4. Find the mean proportional between 10 and 8.

$\frac{10}{x}=\frac{x}{8}$

$x^2=80$

$x=\pm 4\sqrt{5}$

5. Given: $3(x + 2y + 6) = 2(5x + y + 9)$, find the ratio of x to y.

$3x+6y+18=10x+2y+18$

$4y=7x$

$\frac{x}{y}=\frac{4}{7}$

6. Solve for x: $\frac{x+5}{6}=\frac{7}{x-6}$

$x^2-x-30=42$

$x^2-x-72=0$

$(x-9)(x+8)=0$

$x=9,-8$

7. $3\sqrt{6}$ is the mean proportional between 6 and what number?

$\frac{6}{3\sqrt{6}}=\frac{3\sqrt{6}}{x}$

$6x=9\cdot 6$

$x=9$

8. Find the ratio of g to h for $kg-2h=3(mh-5g)$

$kg-2h=3mh-15g$

$kg+15g=2h+3mh$

$g(k+15)=h(2+3m)$

$\frac{g}{h}=\frac{2+3m}{k+15}$