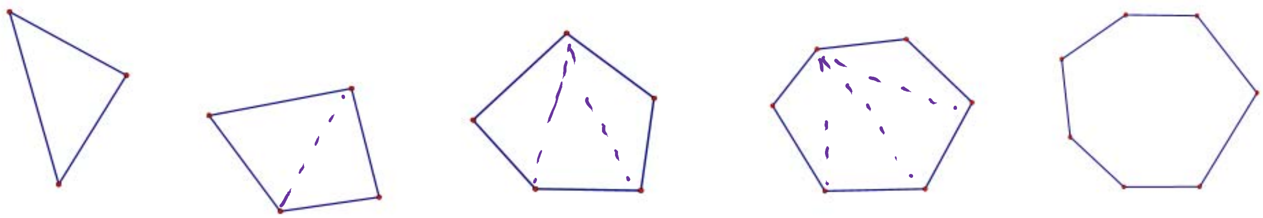
7.3

Monday, November 21, 2016 1:24 PM

7.3 Notes

7.3 – Relationships Involving Polygons

Today's Objective: To identify patterns and discover new formulas that apply to polygons



Polygon Name	# of sides (n)	# of Δs	Sum of interior angles
Triangle	3	1	$180°$
Quad	4	2	$360°$
Pentagon	5	3	$540°$
Hexagon	6	4	$720°$
11-gon	11	9	$1{,}620°$
n - gon	n	$n-2$	$180°(n-2)$

Sum of Exterior Angles *(one per vertex)*
$360°$
$360°$
$360°$
$360°$
$360°$
$360°$

Summarize:

Sum of interior angles of a polygon (n-gon):

$S_i = 180°(n-2)$

Sum of exterior angles of a polygon (n-gon):

$S_e = 360°$

Let's explore the sum of the exterior angles:

Practice:

1) Find the sum of the measures of the interior angles of a decagon.

$S_i = 180°(10-2) = 180°(8) = 1440°$

2) What is the sum of the interior angles of a polygon with 102 sides?

$S_i = 180°(102-2) = 180°(100) = 18{,}000°$

3) Find the sum of the measures of the exterior angles, one per vertex, of a hexagon.

$S_e = 360°$

4) How many sides does a polygon have if the sum of the measures of its angles is 1,620°?

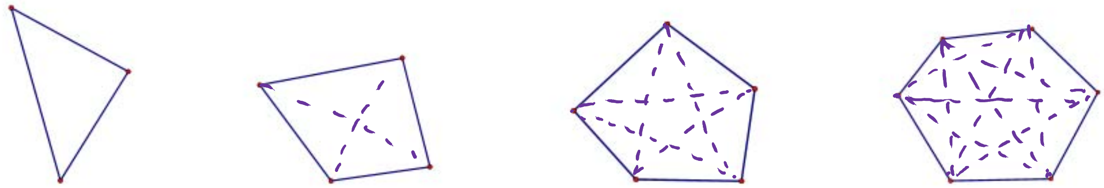
$1{,}620 = 180(n-2)$

$9 = n-2$

$11 = n$

11 sides

Diagonal Formulas!



Shape Name	# of vertices	# of diagonals from a single vertex	Total number of diagonals
triangle	3	0	0
quadrilateral	4	1	2
pentagon	5	2	5
hexagon	6	3	9
n - gon	n	$n-3$	$\frac{n(n-3)}{2}$

Summarize:

Number of diagonals from a single vertex:

$n-3$

Total number of diagonals in a polygon (n-gon):

$d = \frac{n(n-3)}{2}$

Practice:

1) Find the number of diagonals in a pentadecagon.

$$d = \frac{15 \cdot (15-3)}{2} = \frac{15 \cdot 12}{2} = 15 \cdot 6 = 90$$

2) What is the name of the polygon that has 104 diagonals?

$$\frac{n(n-3)}{2} = 104$$

$$n(n-3) = 208$$

$$n^2 - 3n - 208 = 0$$

$$(n+13)(n-16) = 0$$

$$n = \cancel{-13}, 16$$

16 gon

hexa decagon

3) What is the name of the polygon that has 40 diagonals?

$$\frac{n(n-3)}{2} = 40$$

$$n(n-3) = 80$$

$$n^2 - 3n - 80 = 0$$

There is NO polygon.

4) **Always, Sometimes, Never**: The number of diagonals of a polygon is the same as the number of sides.

Sometimes! For a pentagon.

7.3 Practice Problems!

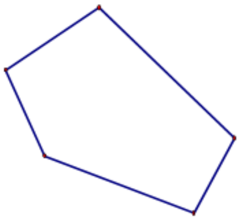
The **sum** of the measures of the **interior angles** of a polygon: $S_i =$ $180°(n-2)$

The **sum** of the measures of the ***exterior angles*** of a polygon: $S_e =$ $360°$

Number of diagonals in a polygon **(from a single vertex):** $D_s =$ $n-3$

The **total number** of diagonals in a polygon: $D =$ $n(n-3)/2$

1) Find the sum of the measure of the



   a. measures of the interior angles of the figure: $S_i = 180°(5-2) = 540°$

   b. measures of the exterior angles of the figure: $S_e = 360°$

2) Find the sum of the measures of the interior angles of a 22-gon.

$S_i = 180°(22-2) = 180°(20) = 3{,}600°$

3) Find the total number of diagonals in a dodecagon.

$d = \frac{12 \cdot 9}{2} = 6 \cdot 9 = 54$

4) Find the sum of the measures of the exterior angles of a hexagon.

$S_e = 360°$

5) How many sides does a polygon have if the sum of the measures of its angles is 2,160°?

$2160 = 180(n-2)$

$12 = n-2$

$n = 14$

6) What is the name of the polygon that has 54 diagonals?

$n(n-3) = 108$

$n^2 - 3n - 108$

$(n-12)(n+9) = 0$

$n = 12, -9$

dodecagon

7) The number of diagonals from a *single vertex* of a 20-gon.

Always, Sometimes, Never

8) As the number of sides of a polygon increases, the sum of the measures of the interior angles of a polygon increases. A

9) As the number of sides of a polygon increases, the number of exterior angles increases. A

10) As the number of sides of a polygon increases, the sum of the measures of the exterior angles increases. N

11) As the number of sides of a polygon increases, the number of diagonals increases. A

12) The sum of the interior angles of a polygon is divisible by 180. A