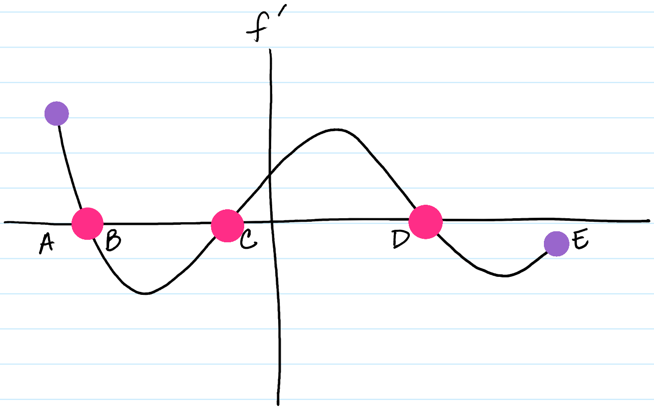
5.3 day 2

Sunday, November 5, 2017 8:15 PM

Return Quiz

Describe the extrema of f given f' below.



There are maxima at B & D because f' changes from (+) to (−).

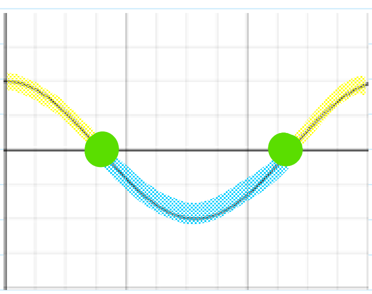
There is a minimum at C because f' changes from (−) to (+)

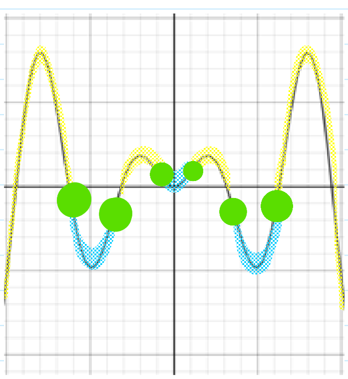
There is a minimum at A because A is a left endpt + f' > 0 on (A, B).

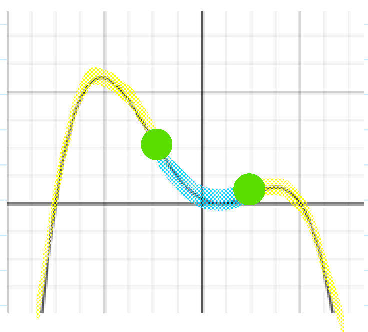
There is a minimum at E because E is a right endpt + f' < 0 on (D, E).

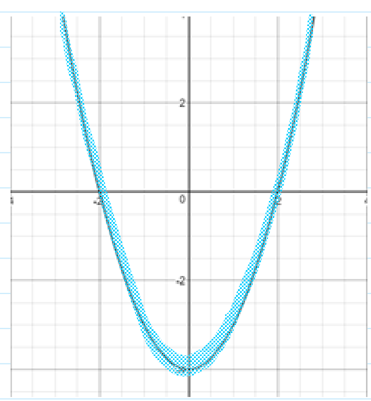
Check HW: subtlety in MVT https://www.geogebra.org/m/YU2ZzC4G

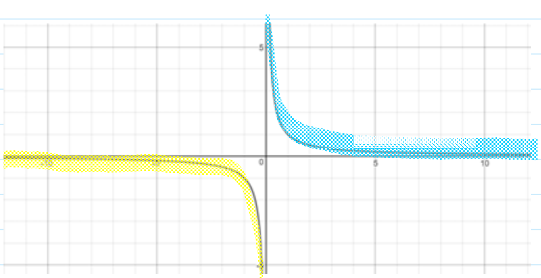
What are we studying today!!?

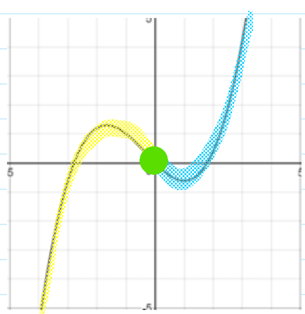












CONCAVITY

Let $f(x)$ be twice differentiable. AKA we can take the derivative two times.

If $f'' > 0$, then $f(x)$ is concave up. holds water 1st derivatives are increasing

If $f'' < 0$, then $f(x)$ is concave down. dumps water 1st derivatives are decreasing

A point of inflection occurs when concavity changes sign.

This can happen when $f'' = 0$ or f'' does NOT exist.

① Let $f(x) = e^{-x^2}$. Find ...

A. point(s) of inflection

B. where $f(x)$ is concave up

C. where $f(x)$ is concave down

$f'(x) = e^{-x^2} \cdot -2x = -2x e^{-x^2}$

$f''(x) = +2x \cdot e^{-x^2} \cdot +2x + e^{-x^2} \cdot -2$

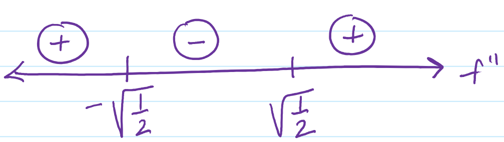
$= 4x^2 e^{-x^2} - 2e^{-x^2}$

$= \underbrace{e^{-x^2}}_{\text{always } +} (4x^2 - 2) = 0$

$4x^2 - 2 = 0$

$x^2 = \frac{1}{2}$

$x = \pm\sqrt{\frac{1}{2}}$



A. There are POI at $\left(-\sqrt{\frac{1}{2}}, \frac{1}{\sqrt{e}}\right)$ and $\left(\sqrt{\frac{1}{2}}, \frac{1}{\sqrt{e}}\right)$ because f'' changes sign.

B. $f(x)$ is concave up on $\left(-\infty, -\sqrt{\frac{1}{2}}\right) \cup \left(\sqrt{\frac{1}{2}}, \infty\right)$ because $f'' > 0$.

C. $f(x)$ is concave down on $\left(-\sqrt{\frac{1}{2}}, \sqrt{\frac{1}{2}}\right)$