5.3 day 2

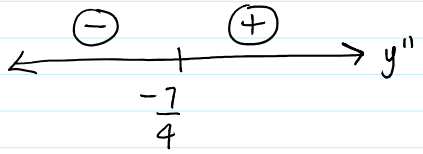
Saturday, October 7, 2017 9:44 PM

pg. 220 #7, 8, 10-13, 15, 18, 22

7. $y = 4x^3 + 21x^2 + 36x - 20$

$y' = 12x^2 + 42x + 36$

$y'' = 24x + 42 = 0$



y is concave down on $(-\infty, -7/4)$ because $y'' < 0$

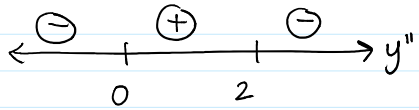
y is concave up on $(-7/4, \infty)$ because $y'' > 0$

8. $y = -x^4 + 4x^3 - 4x + 1$

$y' = -4x^3 + 12x^2 - 4$

$y'' = -12x^2 + 24x$

$= -12x(x - 2) = 0$



y is concave down on $(-\infty, 0) \cup (2, \infty)$ because $y'' < 0$

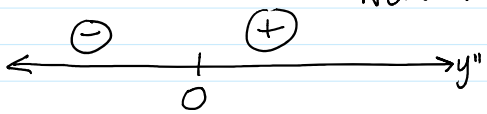
y is concave up on $(0, 2)$ because $y'' > 0$

10. $y = 5 - x^{1/3}$

$y' = -\frac{1}{3} x^{-2/3}$

$y'' = \frac{2}{9} x^{-5/3} = \frac{2}{9x^{5/3}} = 0$

Never! Also $x \neq 0$



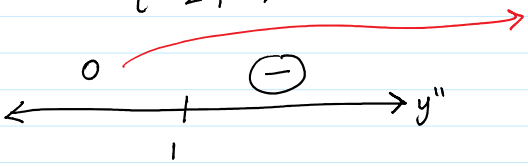
y is concave down on $(-\infty, 0)$ because $y'' < 0$.

y is concave up on $(0, \infty)$ because $y'' > 0$

11. $y = \begin{cases} 2x, & x < 1 \\ 2 - x^2, & x \geq 1 \end{cases}$

$y' = \begin{cases} 2, & x < 1 \\ -2x, & x > 1 \end{cases}$

$y'' = \begin{cases} 0, & x < 1 \\ -2, & x > 1 \end{cases}$



means neither concave up or concave down. LINEAR!

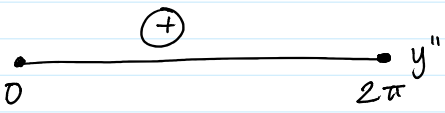
y is concave down on $(1, \infty)$ because $y'' < 0$

y is never concave up because y'' is never > 0.

12. $y = e^x$

$y' = e^x$

$y'' = e^x = 0$



y is concave up on $(0, 2\pi)$ because $y'' > 0$.

y is never concave down because y'' is never < 0.

13. $y = xe^x$

$y' = x \cdot e^x + e^x = e^x(x+1)$

$y'' = e^x \cdot 1 + (x+1) \cdot e^x$

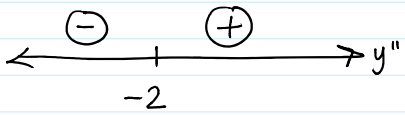
$= e^x(x+2) = 0$

$e^x = 0$ or $x + 2 = 0$

Impossible $x = -2$

$e^x = 0$ or $x+2=0$

Never $x=-2$



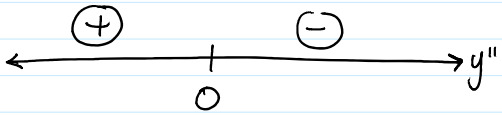
The point $(-2, -2e^{-2})$ is a point of inflection on y because y'' changes from $-$ to $+$.

15. $y = \tan^{-1} x$

$y' = \dfrac{1}{1+x^2} = (1+x^2)^{-1}$

$y'' = -1(1+x^2)^{-2} \cdot 2x$

$= \dfrac{-2x}{(1+x^2)^2} = 0$



y has a point of inflection at $(0,0)$ because y'' changes from $+$ to $-$.

18. $y = x^{1/2}(x+3)$

$y' = x^{1/2} \cdot 1 + (x+3) \cdot \frac{1}{2} x^{-1/2}$

$= \frac{1}{2} x^{-1/2}(2x + x + 3)$

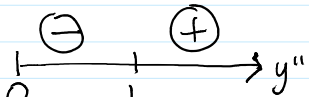
$= \frac{1}{2} x^{-1/2}(3x+3)$

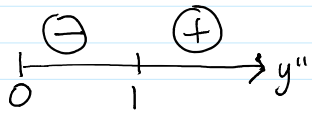
$y'' = \frac{1}{2} x^{-1/2} \cdot 3 + (3x+3) \cdot \frac{1}{2} \cdot -\frac{1}{2} x^{-3/2}$

$= \frac{1}{4} x^{-3/2}[6x - (3x+3)]$

$= \frac{1}{4} x^{-3/2}(3x-3) = 0$

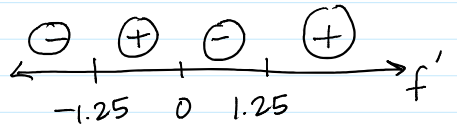
$= \dfrac{3x-3}{4x^{3/2}} = 0$



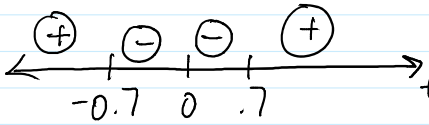


y has a point of inflection at (1, 4) because y" changes from − to +

22.



a. $f' = 0$ at $x = 0, \pm 1.25$
$f' > 0$ for $(-1.25, 0) \cup (1.25, \infty)$
$f' < 0$ for $(-\infty, -1.25) \cup (0, 1.25)$



b. $f'' = 0$ at ± 0.7 changes concavity
$f'' > 0$ for $(-\infty, -0.7) \cup (0.7, \infty)$ concave up
$f'' < 0$ for $(-0.7, 0.7)$ concave down