4.4 day 1

Friday, October 13, 2017 8:26 PM

Two FANCY derivatives that will be with us the rest of the year! Learn to LOVE them!

What is $\frac{d}{dx} e^x$?

$$\frac{d}{dx} e^x = \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \to 0} \frac{e^x \cdot e^h - e^x}{h} = \lim_{h \to 0} e^x \frac{(e^h - 1)}{h} = e^x \cdot 1$$

The derivative of e^x is e^x!

Looks like a great flash card!

Find the derivatives of the following functions. Don't forget chain Rule!

$$\frac{d}{dx} e^x = e^x$$

1. $y = e^{-8x}$

$y' = -8e^{-8x}$

2. $y = e^{\sin x}$

$y' = e^{\sin x} \cdot \cos x$

$= \cos x \left(e^{\sin x}\right)$

3. $y = e^{\sqrt{x}}$

$y' = e^{\sqrt{x}} \cdot \frac{1}{2} x^{-1/2}$

$= \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

4. $y = e^{\sin^{-1} x}$

$y' = e^{\sin^{-1} x} \cdot \frac{1}{\sqrt{1-x^2}}$

$= \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}}$

Since we are talking about derivatives of inverses, let's talk $\frac{d}{dx} \ln x$.

$$y = \ln x$$

$$\frac{d}{dx} e^y = \frac{d}{dx} x$$

$$e^y \cdot \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{e^{\ln x}} = \frac{1}{x}$$

So $\frac{d}{dx} \ln x = \frac{1}{x}$

Looks like a great flash card!

Find the derivatives of the following functions. Don't forget chain rule!

5. $y = \ln(3-5x)$

$y' = \frac{1}{3-5x} \cdot -5$

$= \frac{-5}{3-5x}$

6. $y = \ln \tan x$

$y' = \frac{1}{\tan x} \cdot \sec^2 x$

$= \frac{\sec^2 x}{\tan x}$

7. $y = \ln(6x^2)$

$y' = \frac{1}{6x^2} \cdot 12x$

8. $y = \ln \tan^{-1} x$

$y' = \frac{1}{\tan^{-1} x} \cdot \frac{1}{1+x^2}$

$= \frac{12x}{6x^2}$

$= \frac{2}{x}$

9. What is the domain of y' if $y = \ln(3-5x)$?

$y' = \frac{1}{3-5x} \cdot -5 = \frac{-5}{3-5x}$

$3-5x > 0$

$3 > 5x$

$\frac{3}{5} > x$ WINS. MORE STRICT!

$3-5x \neq 0$

$-5x \neq -3$

$x \neq \frac{3}{5}$

Domain of y' is $(-\infty, 3/5)$

10. Let f + g be inverses of each other.

$f(4) = 10$ $\quad g(10) = 4$

$f'(4) = 3$ $\quad g'(10) = \frac{1}{3}$

Write the equation of a tangent line to g.

$y - 4 = \frac{1}{3}(x-10)$

52 from HW