4.3 day 2

Monday, September 18, 2017 7:08 PM

pg. 178 #1, 6, 13, 14, 17, 18, 21, 23, 28, 29, 39

☆ $\cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$

☆ $\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$

1. $y = \cos^{-1}(x^2)$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-(x^2)^2}} \cdot 2x = \frac{-2x}{\sqrt{1-x^4}}$$

6. $y = s\sqrt{1-s^2} + \cos^{-1}s = s\sqrt{1-s^2} + \underbrace{\pi/2 - \sin^{-1}(s)}_{\cos^{-1}s}$

$$\frac{dy}{ds} = s\cdot\frac{1}{2}(1-s^2)^{-1/2}(-2s) + (1-s^2)^{1/2} - \frac{1}{\sqrt{1-s^2}}$$

$$= -s^2(1-s^2)^{-1/2} + (1-s^2)^{1/2} - (1-s^2)^{-1/2}$$

$$= (1-s^2)^{-1/2}\left(-s^2 + (1-s^2)^{2/2} - 1\right)$$

$$= (1-s^2)^{-1/2}(-2s^2)$$

$$= \frac{-2s^2}{\sqrt{1-s^2}}$$

13. $y = \sec^{-1}(2s+1)$

☆ $\frac{d}{dx}\sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}}$

$$\frac{dy}{ds} = \frac{1}{|2s+1|\sqrt{(2s+1)^2-1}} \cdot 2$$

$$= \frac{2}{|2s+1|\sqrt{4s^2+4s+1-1}}$$

$$= \frac{2}{|2s+1|\sqrt{4s^2+4s}}$$

$$= \frac{2}{|2s+1|\sqrt{4(s^2+s)}}$$

$$= \frac{1}{|2s+1|\sqrt{s^2+s}}$$

$$= \frac{1}{|2s+1|\sqrt{s^2+s}}$$

14. $y = \sec^{-1} 5s$

$*\ \frac{d}{dx} \sec x = \frac{1}{|x|\sqrt{x^2-1}}$

$$\frac{dy}{ds} = \frac{1}{|5s|\sqrt{(5s)^2-1}} \cdot 5$$

$$= \frac{1}{|s|\sqrt{25s^2-1}}$$

17. $y = \sec^{-1}\left(\frac{1}{t}\right)$, $0<t<1$

$$\frac{dy}{dt} = \frac{1}{\left|\frac{1}{t}\right|\sqrt{\left(\frac{1}{t}\right)^2-1}} \cdot -1t^{-2}$$

$$= \frac{-1t^{-2}}{\frac{1}{t}\sqrt{\frac{1}{t^2}-1}}$$

$$= \frac{-1 \cdot t^{-2}}{\frac{1}{t}\sqrt{\frac{1-t^2}{t^2}}}$$

$$= \frac{-1t^{-2}}{\frac{1}{t^2}\sqrt{1-t^2}}$$

$$= \frac{-1}{\sqrt{1-t^2}}$$

18. $y = \cot^{-1}\sqrt{t} = \frac{\pi}{2} - \tan^{-1}\sqrt{t}$

$*\ \frac{d}{dx} \tan^{-1} x = \frac{1}{x^2+1}$

$$\frac{dy}{dt} = -\frac{1}{1+(\sqrt{t})^2} \cdot \frac{1}{2} t^{-1/2} = \frac{-1}{2\sqrt{t}\,(1+t)}$$

18. $y = \cot^{-1}\sqrt{t} = \frac{\pi}{2} - \tan^{-1}\sqrt{t}$

$\frac{dy}{dt} = -\frac{1}{1+(\sqrt{t})^2} \cdot \frac{1}{2}t^{-1/2} = \frac{-1}{2\sqrt{t}\,(1+t)}$

☆ $\frac{d}{dx}\tan^{-1}x = \frac{1}{x^2+1}$

21. $y = \tan^{-1}\sqrt{x^2-1} + \csc^{-1}x$, $x>1$

$= \tan^{-1}\sqrt{x^2-1} + \frac{\pi}{2} - \sec^{-1}x$

$\frac{dy}{dx} = \frac{1}{1+\left(\sqrt{x^2-1}\right)^2} \cdot \frac{1}{2}(x^2-1)^{-1/2} \cdot 2x - \frac{1}{|x|\sqrt{x^2-1}}$

$= \frac{x(x^2-1)^{-1/2}}{1+x^2-1} - \frac{1}{|x|\sqrt{x^2-1}}$

$= \frac{1}{x(x^2-1)^{1/2}} - \frac{1}{x(x^2-1)^{1/2}}$

$= 0$

☆ $\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$

☆ $\csc^{-1}x = \frac{\pi}{2} - \sec^{-1}x$

☆ $\frac{d}{dx}\sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}}$

23. $y = \sec^{-1}x$

$\frac{dy}{dx} = \frac{1}{|x|\sqrt{x^2-1}}$

☆ $\frac{d}{dx}\sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}}$

At $x=2$, $\frac{dy}{dx} = \frac{1}{2\sqrt{4-1}} = \frac{1}{2\sqrt{3}}$

At $x=2$, $y = \sec^{-1}2 = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

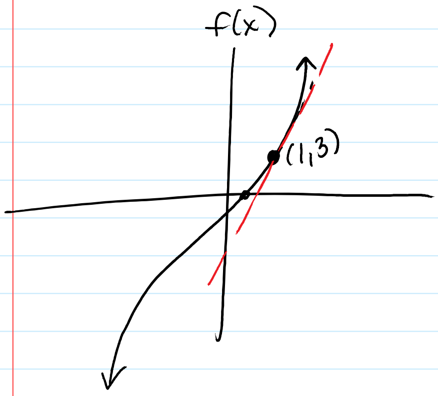
$y - \frac{\pi}{3} = \frac{1}{2\sqrt{3}}(x-2)$

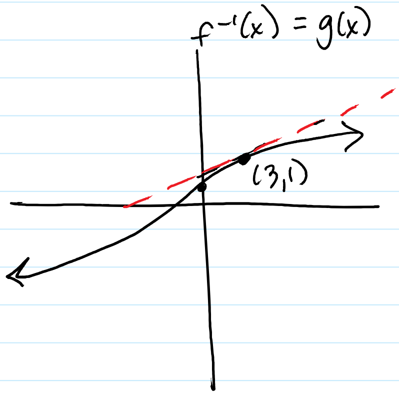
$y = \frac{1}{2\sqrt{3}}x - \frac{2}{2\sqrt{3}} + \frac{\pi}{3}$

$y = 0.289x - 0.470$

28. $f(x) = x^5 + 2x^3 + x - 1$ $\quad f(1) = 1 + 2 + 1 - 1 = 3 \implies g(3) = 1$

$f'(x) = 5x^4 + 6x^2 + 1$ $\quad f'(1) = 5 + 6 + 1 = 12 \implies g'(3) = \frac{1}{12}$





29. $f(x) = \cos x + 3x$

a. f has differentiable inverse means

f is differentiable everywhere AND $f' \neq 0$.

$f'(x) = -\sin x + 3$ exists everywhere and is between $[2, 4]$.

b. $f(0) = \cos 0 + 3 \cdot 0 = 1$

$f'(0) = -\sin 0 + 3 = 3$

c. Let $f^{-1}(x) = g(x)$

$g(1) = 0$ so $g'(1) = \frac{1}{3}$

39. $\frac{d}{dx} \sec^{-1} x^2 = \frac{1}{|x^2|\sqrt{(x^2)^2 - 1}} \cdot 2x = \frac{2x}{x^2\sqrt{x^4-1}} = \frac{2}{x\sqrt{x^4-1}}$

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