4.3 day 1

Monday, September 18, 2017 8:14 AM

pg. 178 #3, 4, 7, 9, 18, 25, 27, 35-38, 40

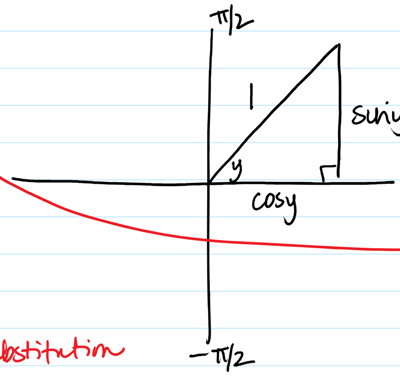
$y = \sin^{-1}(x)$ ★ Know this derivation

$\frac{d}{dx}\sin(y) = \frac{d}{dx}x$

$\cos y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\cos y}$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$ Make Substitution



$\cos^2 y + \sin^2 y = 1$

so $\cos y = \sqrt{1-\sin^2 y}$

But $\sin y = x$.

$\therefore \cos y = \sqrt{1-x^2}$

3. $y = \sin^{-1}(\sqrt{2}t)$

★ $\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$

$\frac{dy}{dt} = \frac{1}{\sqrt{1-(\sqrt{2}t)^2}} \cdot \sqrt{2}$

$= \frac{\sqrt{2}}{\sqrt{1-2t^2}}$

4. $y = \sin^{-1}(1-t)$

$\frac{dy}{dt} = \frac{1}{\sqrt{1-(1-t)^2}} \cdot -1$

$= \frac{-1}{\sqrt{1-(1-2t+t^2)}}$

$= \frac{-1}{\sqrt{-t^2+2t}}$

7. $y = x\sin^{-1}x + \sqrt{1-x^2}$

$\frac{dy}{dx} = x \cdot \frac{1}{\sqrt{1-x^2}} + \sin^{-1}x \cdot 1 + \frac{1}{2}(1-x^2)^{-1/2} \cdot -2x$

$$= x \cdot \frac{1}{\sqrt{1-x^2}} + \sin^{-1}x - x \cdot \frac{1}{\sqrt{1-x^2}}$$

$$= \sin^{-1}x$$

9. $x(t) = \sin^{-1}\left(\frac{t}{4}\right)$

$$x'(t) = v(t) = \frac{1}{\sqrt{1-(t/4)^2}} \cdot \frac{1}{4} = \frac{1}{\sqrt{1-\frac{t^2}{16}}} \cdot \frac{1}{4} = \frac{1}{\sqrt{\frac{16-t^2}{16}}} \cdot \frac{1}{4} = \frac{1}{\sqrt{16-t^2}}$$

$$x'(3) = \frac{1}{\sqrt{16-9}} = \frac{1}{\sqrt{7}} = \frac{\sqrt{7}}{7}$$

25. $y = \sin^{-1}\left(\frac{x}{4}\right)$

★ $\frac{d}{dx} \sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\frac{x}{4}\right)^2}} \cdot \frac{1}{4}$$

$$= \frac{1}{4\sqrt{1-\frac{x^2}{16}}}$$

$$= \frac{1}{4\sqrt{\frac{16-x^2}{16}}}$$

$$= \frac{1}{\sqrt{16-x^2}}$$

At $x=3$, $\frac{dy}{dx} = \frac{1}{\sqrt{16-9}} = \frac{1}{\sqrt{7}}$

At $x=3$, $y = \sin^{-1}\left(\frac{3}{4}\right) = 0.848$

$x=3,\ y=\sin^{-1}(\frac{3}{4})=0.848$

$$y-0.848=\frac{1}{\sqrt{7}}(x-3)$$

$$y=0.378x-0.286$$

27. a. $y=\tan x$ $\qquad$ y @ $x=\pi/4$ is 1

$$\frac{dy}{dx}=\sec^2 x$$

Tangent line: $y-1=2(x-\pi/4)$

$$\left.\frac{dy}{dx}\right|_{x=\pi/4}=\left(\frac{2}{\sqrt{2}}\right)^2=2$$

b. $y=\tan^{-1}(x)$ $\qquad$ y @ $x=1$ is $\pi/4$

$$\frac{dy}{dx}=\frac{1}{1+x^2}$$

Tangent line: $y-\frac{\pi}{4}=\frac{1}{2}(x-1)$

$$\left.\frac{dy}{dx}\right|_{x=1}=\frac{1}{1+1}=\frac{1}{2}$$

35. True . $\sin x$ is bounded

36. False . $\tan x$ is not bounded

37. $$\frac{d}{dx}\sin^{-1}\left(\frac{x}{2}\right)=\frac{1}{\sqrt{1-(\frac{x}{2})^2}}\cdot\frac{1}{2}=\frac{1}{2\sqrt{1-\frac{x^2}{4}}}=\frac{1}{2\sqrt{\frac{4-x^2}{4}}}=\frac{1}{\sqrt{4-x^2}}$$

E

38. $$\frac{d}{dx}\tan 3x=\frac{1}{1+(3x)^2}\cdot 3=\frac{3}{1+9x^2}$$ D

40. $u=\tan^{-1}2x$

$$\frac{dy}{dx} = \frac{1}{1+(2x)^2} \cdot 2 = \frac{2}{1+4x^2}$$

At $x = 1$, $\frac{dy}{dx} = \frac{2}{1+4} = \frac{2}{5}$

C