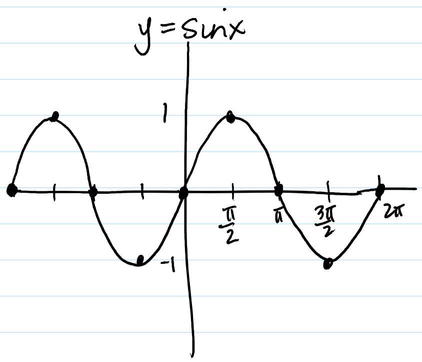
4.3 day 1

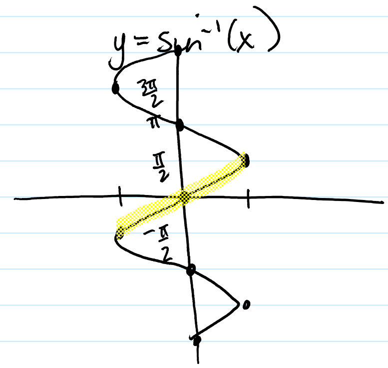
Tuesday, October 10, 2017 10:13 AM

Return tests, quizzes

Go over 4.3 pre-assignment

4.3 is about derivatives of universe functions. We begin with the derivative of $y = \sin^{-1}(x)$





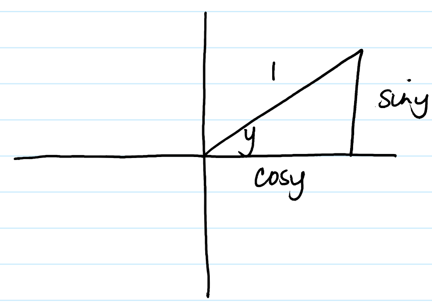
So what's $\frac{d}{dx}\left[y = \sin^{-1}(x)\right]$?

Rewrite as $x = \sin y$.

$$\frac{d}{dx} x = \frac{d}{dx} \sin y$$

$$1 \frac{dx}{dx} = \cos y \cdot \frac{dy}{dx}$$

$$\frac{1}{\cos y} = \frac{dy}{dx}$$



$$\cos^2 y + \sin^2 y = 1$$

$$\cos^2 y = 1 - \sin^2 y$$

$$\cos y = \sqrt{1 - \sin^2 y}$$

BUT WAIT! $\sin y = x$

so $\cos y = \sqrt{1-x^2}$

Therefore, $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

The derivative of $y = \sin^{-1}(x)$ is $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

Looks like a great flash card!

Find the derivative of the following functions. Don't forget Chain Rule!

① $y = \sin^{-1}(3x)$

$$y' = \frac{1}{\sqrt{1-(3x)^2}} \cdot 3$$

$$= \frac{3}{\sqrt{1-9x^2}}$$

② $y = \sin^{-1}(2+5x)$

$$y' = \frac{1}{\sqrt{1-(5x+2)^2}} \cdot 5$$

$$= \frac{5}{\sqrt{-25x^2-20x-3}}$$

③ $y = \sin^{-1}(4x^2)$

$$y' = \frac{1}{\sqrt{1-(4x^2)^2}} \cdot 8x$$

$$= \frac{8x}{\sqrt{1-16x^4}}$$

④ $y = \sin^{-1}(\sqrt{5} \cdot x)$

$$y' = \frac{1}{\sqrt{1-(\sqrt{5}x)^2}} \cdot \sqrt{5}$$

$$= \frac{\sqrt{5}}{\sqrt{1-5x^2}}$$

⑤ $y = \sin^{-1}\left(\frac{1}{x}\right), x > 0$

$$y' = \frac{1}{\sqrt{1-\left(\frac{1}{x}\right)^2}} \cdot -1x^{-2}$$

$$= \frac{-1}{x^2\sqrt{1-\frac{1}{x^2}}}$$

$$= \frac{-1}{x^2\sqrt{\frac{x^2-1}{x^2}}}$$

$$= \frac{-1}{x\sqrt{x^2-1}}$$

⑥ $y = \sin^{-1}\left(\frac{x}{2}\right)$

$$y' = \frac{1}{\sqrt{1-\frac{x^2}{4}}} \cdot \frac{1}{2}$$

$$= \frac{1}{2\sqrt{\frac{4-x^2}{4}}}$$

$$= \frac{1}{\sqrt{4-x^2}}$$

The derivative of $y = \tan^{-1}(x)$ is $\frac{dy}{dx} = \frac{1}{1+x^2}$.

Looks like a great flash card!

Find the derivative of the following functions. Don't forget Chain Rule!

(7) $y = \tan^{-1}(4x)$

$$y' = \frac{1}{1+(4x)^2} \cdot 4$$

$$= \frac{4}{1+16x^2}$$

(8) $y = \tan^{-1}(\sqrt{6} \cdot x)$

$$y' = \frac{1}{1+(\sqrt{6}x)^2} \sqrt{6}$$

$$= \frac{\sqrt{6}}{1+6x^2}$$

(9) $y = \tan^{-1}(7-5x)$

$$y' = \frac{1}{1+(7-5x)^2} \cdot -5$$

$$= \frac{-5}{25x^2-70x+50}$$

$$= \frac{-1}{5x^2-14x+10}$$

(10) $y = \tan^{-1}(x^3)$

$$y' = \frac{1}{1+(x^3)^2} \cdot 3x^2$$

$$= \frac{3x^2}{1+x^6}$$

(11) $y = \tan^{-1}\left(\frac{x}{3}\right)$

$$y' = \frac{1}{1+\left(\frac{x}{3}\right)^2} \cdot \frac{1}{3}$$

$$= \frac{1}{1+\frac{x^2}{9}} \cdot \frac{1}{3}$$

$$= \frac{1}{\frac{9+x^2}{9}} \cdot \frac{1}{3}$$

$$= \frac{9}{9+x^2} \cdot \frac{1}{3}$$

$$= \frac{3}{9+x^2}$$

(12) $y = \tan^{-1}\left(\frac{3}{x}\right)$

$$y' = \frac{1}{1+\left(\frac{3}{x}\right)^2} \cdot -3x^{-2}$$

$$= \frac{1}{1+\frac{9}{x^2}} \cdot -3x^{-2}$$

$$= \frac{1}{\frac{x^2+9}{x^2}} \cdot \frac{-3}{x^2}$$

$$= \frac{-3}{x^2+9}$$