4.2 day 1

Sunday, September 17, 2017 8:26 PM

pg. 170 #1,3,5,8,15,16,19,21

1. $x^2y + xy^2 = 6$

Take the derivative with respect to x for all 3 terms

$$x^2 \cdot \frac{dy}{dx} + y \cdot 2x \frac{dx}{dx} + x \cdot 2y \frac{dy}{dx} + y^2 \cdot \frac{dx}{dx} = 0 \frac{dx}{dx}$$

$$\frac{dy}{dx}\left(x^2 + 2xy\right) + 2xy + y^2 = 0$$

$$\frac{dy}{dx} = \frac{-2xy - y^2}{x^2 + 2xy}$$

3. $y^2 = \frac{x-1}{x+1}$

$$2y \frac{dy}{dx} = \frac{(x+1) \cdot 1 \cdot \frac{dx}{dx} - (x-1) \cdot 1 \cdot \frac{dx}{dx}}{(x+1)^2}$$

$$2y \frac{dy}{dx} = \frac{2}{(x+1)^2}$$

$$\frac{dy}{dx} = \frac{1}{y(x+1)^2}$$

5. $x = \tan y$

$$1 \frac{dx}{dx} = \sec^2 y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y}$$

$$\frac{dy}{dx} = \cos^2 y$$

8. $x + \sin y = xy$

$$1\frac{dx}{dx} + \cos y \cdot \frac{dy}{dx} = x \cdot 1\frac{dy}{dx} + y \cdot 1\frac{dx}{dx}$$

$$1 - y = x\frac{dy}{dx} - \cos y \frac{dy}{dx}$$

$$1 - y = \frac{dy}{dx}\left(x - \cos y\right)$$

$$\frac{dy}{dx} = \frac{1-y}{x-\cos y} = \frac{-1\cdot(1-y)}{-1(x-\cos y)} = \frac{y-1}{\cos y - x}$$

9. $x^2 + y^2 = 13$

$$2x\frac{dx}{dx} + 2y\frac{dy}{dx} = 0\frac{dx}{dx}$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\frac{dy}{dx}\Big|_{(-2,3)} = -\frac{-2}{3} = \frac{2}{3}$$

12. $(x+2)^2 + (y+3)^2 = 25$

$$2(x+2)\cdot 1\cdot\frac{dx}{dx} + 2(y+3)\cdot 1\frac{dy}{dx} = 0\frac{dx}{dx}$$

$$(2y+6)\frac{dy}{dx} = -2x - 4$$

$$\frac{dy}{dx} = \frac{-2x-4}{2y+6}$$

$$\frac{dy}{dx}\Big|_{(1,-7)} = \frac{-2-4}{2\cdot -7+6} = \frac{-6}{-8} = \frac{3}{4}$$

15. $x^3 + y^3 = xy$

15. $x^3 + y^3 = xy$

$3x^2 \cdot \frac{dx}{dx} + 3y^2 \cdot \frac{dy}{dx} = x \cdot 1 \cdot \frac{dy}{dx} + y \cdot 1 \cdot \frac{dx}{dx}$

$3y^2 \frac{dy}{dx} - x\frac{dy}{dx} = y - 3x^2$

$\frac{dy}{dx}(3y^2 - x) = y - 3x^2$

$\frac{dy}{dx} = \frac{y - 3x^2}{3y^2 - x}$ ← problem! $3y^2 - x = 0$

$3y^2 = x$

Defined everywhere except when $y^2 = \frac{x}{3}$

16. $x^2 + 4xy + 4y^2 - 3x = 6$

$2x \cdot \frac{dx}{dx} + 4x \cdot 1 \cdot \frac{dy}{dx} + y \cdot 4 \cdot \frac{dx}{dx} + 8y \cdot \frac{dy}{dx} - 3 \cdot \frac{dx}{dx} = 0 \frac{dx}{dx}$

$\frac{dy}{dx}(4x + 8y) = 3 - 4y - 2x$

$\frac{dy}{dx} = \frac{3 - 4y - 2x}{4x + 8y}$ ← problem! $4x + 8y = 0$

$8y = -4x$

Defined everywhere except when $y = -\frac{1}{2}x$

19. $x^2y^2 = 9$

$x^2 \cdot 2y\frac{dy}{dx} + y^2 \cdot 2x\frac{dx}{dx} = 0\frac{dx}{dx}$

$2x^2y\frac{dy}{dx} = -2xy^2$

$\frac{dy}{dx} = \frac{-2xy^2}{2x^2y}$

$\frac{dy}{dx} = \frac{-y}{x}$

Tangent line: $y - 3 = 3(x+1)$

Normal line: $y - 3 = \frac{-1}{3}(x+1)$

$\frac{dy}{dx}$ $\frac{-3}{x}$

$\left.\frac{dy}{dx}\right|_{(-1,3)} = \frac{-3}{-1} = 3$

21. $6x^2 + 3xy + 2y^2 + 17y - 6 = 0$

$12x\frac{dx}{dx} + 3x\cdot 1\frac{dy}{dx} + y\cdot 3\frac{dx}{dx} + 4y\frac{dy}{dx} + 17\frac{dy}{dx} + 0\frac{dy}{dx} = 0$

$\frac{dy}{dx}(3x + 4y + 17) = -12x - 3y$

$\frac{dy}{dx} = \frac{-12x - 3y}{3x + 4y + 17}$

$\left.\frac{dy}{dx}\right|_{(-1,0)} = \frac{+12}{-3 + 17} = \frac{12}{14} = \frac{6}{7}$

Tangent line: $y = \frac{6}{7}(x+1)$

Normal line: $y = \frac{-7}{6}(x+1)$