AP Calculus AB **Name:**
4.1 day 1

1. Evaluate the following given $f(x) = \sin x$.
   a. $f(4x)$
   $\sin 4x$
   b. $f(2x-1)$
   $\sin(2x-1)$

2. Evaluate the following given $f(x) = x^2$.
   a. $f(\tan x)$
   $(\tan x)^2$
   $= \tan^2 x$
   b. $f\left(\frac{1}{x}\right)$
   $= \left(\frac{1}{x}\right)^2 = \frac{1}{x^2}$

3. Evaluate $f(g(x))$ for $f(x) = x + 7$ and $g(x) = x^{\frac{4}{5}}$.
   $f(g(x)) = x^{4/5} + 7$

4. Evaluate $f(g(x))$ for $f(x) = (x-5)^{17}$ and $g(x) = 8x$.
   $f(g(x)) = (8x-5)^{17}$

5. Evaluate $f(g(x))$ for $f(x) = \cos x$ and $g(x) = \sqrt{x+9}$.
   $f(g(x)) = \cos\sqrt{x+9}$

Write $f(g(x))$ as a composition of two functions by identifying $f(x)$ and $g(x)$. Neither $f(x)$ *nor* $g(x)$ can equal $\pm x$.

1. $f(g(x)) = \sin(5x)$

$f(x) = \sin x$

$g(x) = 5x$

2. $f(g(x)) = (2x+4)^6$

$f(x) = x^6$

$g(x) = 2x+4$

3. $f(g(x)) = (3x+4)^8$

$f(x) = x^8$

$g(x) = 3x+4$

4. $f(g(x)) = (x^4-5)^{-2}$

$f(x) = x^{-2}$

$g(x) = x^4-5$

5. $f(g(x)) = \dfrac{2}{(5x-7)^4}$

$f(x) = \dfrac{2}{x^4}$

$g(x) = 5x-7$

6. $f(g(x)) = (11x-90)^{-3}$

$f(x) = x^{-3}$

$g(x) = 11x-90$

7. $f(g(x)) = \sin^2 x = (\sin(x))^2$

$f(x) = x^2$

$g(x) = \sin x$

8. $f(g(x)) = \tan(\sec x)$

$f(x) = \tan x$

$g(x) = \sec x$

Use the Photomath app to find the derivative $\frac{d}{dx} f(g(x))$ of each composition of functions.

1. $f(g(x)) = \sin(5x)$

$\frac{d}{dx}\sin(5x) =$ $5\cos 5x$

2. $f(g(x)) = (2x+4)^6$

$\frac{d}{dx}(2x+4)^6 =$ $6(2x+4)^5 \cdot 2$

$= 12(2x+4)^5$

3. $f(g(x)) = (3x+4)^8$

$\frac{d}{dx}(3x+4)^8 =$ $8(3x+4)^7 \cdot 3$

$= 24(3x+4)^7$

4. $f(g(x)) = \cos(\sin x)$

$\frac{d}{dx}\cos(\sin x) =$ $-\sin(\sin x)\cdot\cos x$

Take a guess at the derivatives of the compositions below.

5. $\frac{d}{dx}\cos 4x$ $= -4\sin 4x$

6. $\frac{d}{dx}(5x-4)^3$

$= 3(5x-4)^2 \cdot 5$

$= 15(5x-4)^2$

7. $\frac{d}{dx}\tan(\sec x)$

$= \sec^2(\sec x)\cdot \sec x \tan x$

8. Let u be a function of x.

$\frac{d}{dx} f(u) =$ $f'(u)\cdot u'$

The Chain Rule:

$\frac{d}{dx} f(u) = f'(u) \cdot u'$ where u is a function

Find the derivatives of the following functions.

1. $f(x) = \cos(5x+8)$ $f'(x) = -\sin(5x+8) \cdot 5 = -5\sin(5x+8)$

2. $f(x) = \sec 2x$ $f'(x) = \sec 2x \tan 2x \cdot 2$

3. $f(x) = \sin(x^3 + x)$ $f'(x) = \cos(x^3+x) \cdot (3x^2+1)$

4. $f(x) = (4x+9)^5$ $f'(x) = 5(4x+9)^4 \cdot 4 = 20(4x+9)^4$

5. $f(x) = (9-7x)^{-4}$ $f'(x) = -4(9-7x)^{-5} \cdot -7 = 28(9-7x)^{-5}$

6. $f(x) = \tan\sqrt{x}$ $f'(x) = \sec^2\sqrt{x} \cdot \frac{1}{2}x^{-1/2} = \frac{\sec^2\sqrt{x}}{2\sqrt{x}}$

7. $f(x) = 7\cot\left(\frac{3}{x}\right)$ $f'(x) = -7\csc^2\left(\frac{3}{x}\right) \cdot -3x^{-2} = \frac{21\csc^2\left(\frac{3}{x}\right)}{x^2}$

8. $f(x) = (8x + \sqrt[3]{x})^{-4}$ $f'(x) = -4(8x+\sqrt[3]{x})^{-5} \cdot (8 + \frac{1}{3}x^{-2/3})$

#5 from homework $f(x) = \left(\frac{\sin x}{1+\cos x}\right)^2$