3.3 day 1

Friday, September 1, 2017 9:26 PM

pg. 126 #1-11 odd, 25-30 , 32, 33, 35, 37

1. $y = -x^2 + 3$

$\frac{dy}{dx}$ = the derivative of y with respect to x

$= -2x + 0$

$= -2x$

3. $y = 2x + 1$

$\frac{dy}{dx} = 2 + 0$

$= 2$

5. $y = \frac{x^3}{3} + \frac{x^2}{2} + x$

$= 3 \cdot \frac{x^2}{3} + 2\frac{x}{2} + 1$

$= x^2 + x + 1$

7. Horizontal tangent $\Rightarrow$ $\frac{dy}{dx} = f'(x) = 0$

$y = x^3 - 2x^2 + x + 1$

$\frac{dy}{dx} = 3x^2 - 2 \cdot 2x + 1 + 0$

$= 3x^2 - 4x + 1$

Find when $\frac{dy}{dx} = 0$

$3x^2 - 4x + 1 = 0$

$(3x - 1)(x - 1) = 0$

$x = \frac{1}{3}, 1$

9. $y = x^4 - 4x^2 + 1$

$\frac{dy}{dx} = 4x^3 - 4 \cdot 2x + 0$

$= 4x^3 - 8x$

Find when $\frac{dy}{dx} = 0$

$4x^3 - 8x = 0$

$4x(x^2 - 2) = 0$

$4x = 0 \quad \text{OR} \quad x^2 - 2 = 0$

$x = 0, \quad x = \pm\sqrt{2}$

11. $y = 3x^5 - 5x^3$

$\frac{dy}{dx} = 3 \cdot 5x^4 - 5 \cdot 3x^2$

$= 15x^4 - 15x^2$

Find when $\frac{dy}{dx} = 0$

$15x^4 - 15x^2 = 0$

$15x^2(x^2 - 1) = 0$

$15x^2(x-1)(x+1) = 0$

$x = 0, \pm 1$

25. $y = x^2 + 5x$

$\frac{dy}{dx}$ = slope of the tangent line

$\frac{dy}{dx} = 2x + 5$

$\left.\frac{dy}{dx}\right|_{x=3} = 2 \cdot 3 + 5 = 11$

(iii)

26. $3x - 2y + 12 = 0$

$-2y = -12 - 3x$

(...)

da $3x - 2y + 12 = 0$

$-2y = -12 - 3x$

$y = 6 + 3/2x$

(iii)

27. $y = \frac{x^3 + 1}{2x} = \frac{x^3}{2x} + \frac{1}{2x} = \frac{1}{2}x^2 + \frac{1}{2}x^{-1}$

$\frac{dy}{dx} = \frac{1}{2} \cdot 2x + \frac{1}{2} \cdot -1 \cdot x^{-2}$

$= x - \frac{1}{2}x^{-2}$

At $x = 1$, $\frac{dy}{dx} = 1 - \frac{1}{2} \cdot 1^{-2} = \frac{1}{2}$

At $x = 1$, $y = 1$

Equation of tangent line is $y - 1 = \frac{1}{2}(x - 1)$

28. $y = \frac{x^4 + 2}{x^2} = x^2 + 2x^{-2}$

$\frac{dy}{dx} = 2x + 2 \cdot -2x^{-3} = 2x - \frac{4}{x^3}$

At $x = -1$, $\frac{dy}{dx} = 2 \cdot -1 - \frac{4}{(-1)^3} = -2 - \frac{4}{-1} = 2$

At $x = -1$, $y = 3$)

Equation of tangent line is $y - 3 = 2(x + 1)$

29. $y = 4x^{-2} - 8x + 1$

$$\frac{dy}{dx} = 4 \cdot -2x^{-3} - 8$$

$$= -8x^{-3} - 8$$

30. $$y = \frac{x^{-4}}{4} - \frac{x^{-3}}{3} + \frac{x^{-2}}{2} - x^{-1} + 3$$

$$\frac{dy}{dx} = -4 \cdot \frac{x^{-5}}{4} - \frac{-3 \cdot x^{-4}}{3} + \frac{-2 \cdot x^{-3}}{2} - -1 \cdot x^{-2} + 0$$

$$= -x^{-5} + x^{-4} - x^{-3} + x^{-2}$$

32. $$y = 2\sqrt{x} - \frac{1}{\sqrt{x}}$$

$$= 2x^{1/2} - x^{-1/2}$$

$$\frac{dy}{dx} = 2 \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}} - \frac{-1}{2} \cdot x^{-\frac{3}{2}}$$

$$= x^{-1/2} + \frac{1}{2} x^{-\frac{3}{2}}$$

$$= \frac{1}{\sqrt{x}} + \frac{1}{2\sqrt{x^3}}$$

33. $$y = x^4 + x^3 - 2x^2 + x - 5$$

$$y' = 4x^3 + 3x^2 - 2 \cdot 2x + 1 - 0$$

$$= 4x^3 + 3x^2 - 4x + 1$$

$$y'' = 4 \cdot 3x^2 + 3 \cdot 2x - 4 + 0$$

$$= 12x^2 + 6x - 4$$

$$y''' = 12 \cdot 2x + 6 - 0$$

$$= 24x + 6$$

35. $$y = x^{-1} + x^2$$

$$y' = -1 \cdot x^{-2} + 2x$$

$y'' = -1 \cdot -2x^{-3} + 2$

$= 2x^{-3} + 2$

$y''' = 2 \cdot -3x^{-4} + 0$

$= -6x^{-4}$

$= \frac{-6}{x^4}$

36. $y = \frac{x+1}{x} = 1 + \frac{1}{x} = 1 + x^{-1}$

$y' = 0 + -1x^{-2} = -1x^{-2}$

$y'' = -1 \cdot -2 \cdot x^{-3} = 2x^{-3}$

$y''' = 2 \cdot -3 \cdot x^{-4} = -6x^{-4}$

37. $y = x^3 - 3x + 1$

$\frac{dy}{dx} = 3x^2 - 3 + 0$

$= 3x^2 - 3$

$\frac{dy}{dx}\Big|_{x=2} = 3 \cdot 2^2 - 3 = 12 - 3 = 9$ = slope of tangent

$y - 3 = \frac{-1}{9}(x - 2)$ AKA the normal line