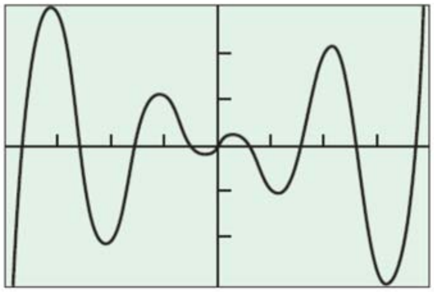
AP Calculus AB **Name:**

3.2 Practice

The graph of a function over a closed interval D is given. At what domain points does the function appear to be

a) Differentiable?
b) Continuous but not differentiable?
c) Neither continuous nor differentiable?
d) If possible, call the situation by a special name.



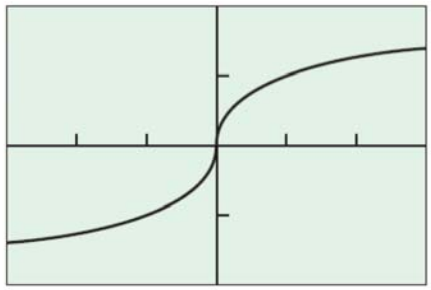
[–4, 4] by [–3, 3]

a) $[-4,4]$

b) None

c) None

d) None



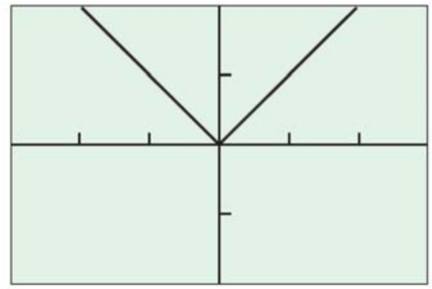
[–3, 3] by [–2, 2]

a) $[-3,0)\cup(0,3]$

b) $x=0$

c) None

d) vertical tangent at $x=0$



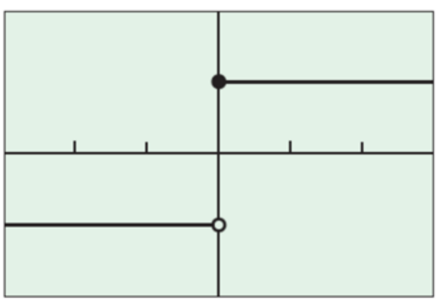
[–3, 3] by [–2, 2]

a) $[-2,0)\cup(0,2]$

b) $x=0$

c) None

d) Corner @ $x=0$



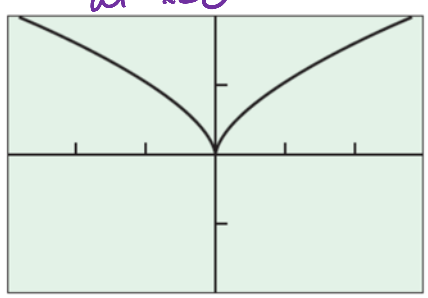
[–3, 3] by [–2, 2]

a) $[-3,0)\cup(0,3]$

b) None

c) $x=0$

d) jump discontinuity at $x=0$



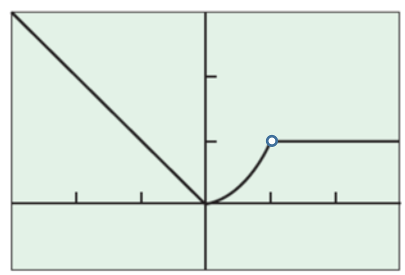
[–3, 3] by [–2, 2]

a) $[-3,0)\cup(0,3]$

b) $x=0$

c) None

d) cusp @ $x=0$



[–3, 3] by [–1, 3]

a) $[-3,0)\cup(0,1)\cup(1,3]$

b) $x=0$

c) $x=1$

d) Corner @ $x=0$ removable discontinuity @ $x=1$

2. Find the unique value k that makes $f(x)=\begin{cases}-4x+k & x<-2 \\ x^2 & x\geq -2\end{cases}$ differentiable at x = -2.

① Need continuity first

$-4(-2)+k=(-2)^2 \implies k=-4$

$\boxed{k=-4}$

② Now check differentiability

$f'(-2)$

Left $=-4$ Right $=-4$

$f'(-2^+)=\lim_{h\to 0}\frac{(-2+h)^2-4}{h}=\lim_{h\to 0}\frac{4-4h+h^2-4}{h}$

$=-4$

3. Show that the derivative of $f(x)=3x^2+x$ has a zero in [-2,5].

$$f'(x)=\lim_{h\to 0}\frac{3(x+h)^2+x+h-(3x^2+x)}{h}$$

$$=\lim_{h\to 0}\frac{3(x^2+2hx+h^2)+x+h-3x^2-x}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{3x^2}+6hx+3h^2+\cancel{x}+h-\cancel{3x^2}-\cancel{x}}{h}$$

$$=\lim_{h\to 0}\frac{6hx+3h^2+h}{h}$$

$$=\lim_{h\to 0} 6x+3h+1$$

$$=6x+1$$

$f'(x)=6x+1$ is continuous

$f'(-2)=-11$

$f'(5)=31$

Since $-11<0<31$, there exists a zero in [-2,5].