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29. A of circle = πr^2

$$\begin{aligned} \text{instantaneous ROC @ } r=3 &= \lim_{h \rightarrow 0} \frac{\pi(3+h)^2 - \pi \cdot 3^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{\pi(9 + 6h + h^2) - 9\pi}{h} \\ &= \lim_{h \rightarrow 0} \frac{\pi h^2 + 6\pi h}{h} \\ &= \lim_{h \rightarrow 0} \pi h + 6\pi \\ &= 6\pi \frac{h^2}{h} \end{aligned}$$

33. $f(x) = x^2 + 4x - 1$ horizontal tangent \Rightarrow slope = 0

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = 0$$

$$= \lim_{h \rightarrow 0} \frac{(x+h)^2 + 4(x+h) - 1 - [x^2 + 4x - 1]}{h} = 0$$

$$= \lim_{h \rightarrow 0} \frac{x^2 + 2hx + h^2 + 4x + 4h - 1 - x^2 - 4x + 1}{h} = 0$$

$$= \lim_{h \rightarrow 0} \frac{2hx + 4h + h^2}{h} = 0$$

$$= \lim_{h \rightarrow 0} 2x + 4 + h = 0$$

$$2x + 4 = 0$$

$$2x = -4$$

$$x = -2 \quad (\text{now plug in to } f(x))$$

The tangent is horizontal at $f(-2) = -5$

40. Average ROC \Rightarrow simple slope
NO difference quotient

A. 3.4 BLN \$/yr

B. 30 BLN \$/yr

C. -36.5 BLN \$/yr

D. It's all over the place and significantly!

41. TRUE

42. FALSE

43. D

44. E

45. C

46. A

Have to consider left + right slopes and they don't equal

$$\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{2 - 2}{1+h} - 2$$

normal is \perp to tangent
 $y - 2 = \frac{1}{2}(x - 1)$

$$= \lim_{h \rightarrow 0} \frac{2 - 2 - 2h}{n(1+h)}$$

$$= \lim_{h \rightarrow 0} \frac{-2h}{n(1+h)}$$

$$= \lim_{h \rightarrow 0} \frac{-2}{1+h}$$

$$= -2 \quad \leftarrow \text{slope of tangent and through } f(1) = 2$$

$$y - 2 = -2(x - 1)$$

AP Prep

1. D

2. E

3. B

$$\rightarrow \begin{matrix} f(0) = 1 \\ f(3) = 2 \end{matrix} \quad \text{Avg. ROC} = \frac{2-1}{3-0} = \frac{1}{3}$$

$$\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{9 - (2+h)^2 - (9-4)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{9 - 4 - 4h + h^2 - 5}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h^2 - 4h}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{h}(h-4)}{\cancel{h}}$$

$$= -4 \quad \text{Slope of tangent and} \\ \text{through } (2, 5) \\ y - 5 = -4(x - 2)$$

4. $f(x) = 2x - x^2$

a. $f(3) = -3$

b. $f(3+h) = 2(3+h) - (3+h)^2$
 $= 6 + 2h - (9 + 6h + h^2)$
 $= 6 + 2h - 9 - 6h - h^2$
 $= -h^2 - 4h - 3$

c. $\frac{f(3+h) - f(3)}{h} = \frac{-h^2 - 4h - 3 + 3}{h}$
 $= \frac{-h^2 - 4h}{h}$
 $= -h - 4$

d. $\lim_{h \rightarrow 0} -h - 4 = -4$