11.3 A and G sequences

Wednesday, April 19, 2017 8:09 AM

Algebra 2 Trig H **Name:**
11.3 Special types of sequences

Complete the blanks and/or complete the tables in each problem.

1)

Term #	How many black triangles?
1	1
2	3
3	9
4	27
5	81

Exp

2) Linear

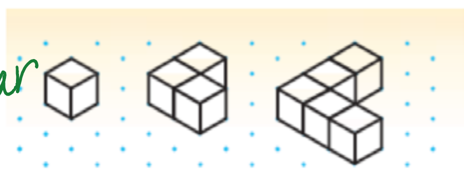

Figure #	Volume
1	1
2	3
3	5
4	7
5	9

3) -4, -2, 0, 2, 4, 6, 8

Linear

4) 4, -2, 1, -½, 1/4, -1/8, 1/16 Exponential

5) 8, 15, 22, 29, 36, 43, 50

Linear

6) 4, 9, 16, 25 36, 49, 64 Quadratic

Definitions:

Arithmetic sequences: constant addition (linear)

Geometric sequences: constant multiplication (exponential)

Arithmetic sequence formulas and their derivations:

Term	55	49	43	37	…	
Symbols	a_1	a_2	a_3	a_4	…	a_n
Term number	1	2	3	4	…	n
Process	$55+-6(0)$	$55 + -6(1)$	$55+-6(2)$	$55+-6(3)$	…	$55+-6(n-1)$

General explicit formula:

$a_n = a_1 + d(n-1)$

Recursive formula:

$$\begin{cases} a_1 = \text{start} \\ a_n = a_{n-1} + d, \text{ for } n \geq 2 \end{cases}$$

Let a_n be an arithmetic sequence.

7. $a_1 = 25$, $a_2 = 16$

a. Find d.

$d = 25-16 = 9$

$d = 16-25 = -9$

b. Write a recursive formula for a_n.

$$\begin{cases} a_1 = 25 \\ a_n = a_{n-1} - 9, \text{ for } n \geq 2 \end{cases}$$

c. Write an explicit formula for a_n.

$a_n = 25 + -9(n-1)$

d. Find a_{15}.

$a_{15} = 25+-9(15-1)$

$= 25 + -9 \cdot 14$

$= 25 - 126$

$= -101$

8. $a_1 = 36$, $a_7 = 24$

a. Find d.

$24 = 36 + d(7-1)$

$-12 = d \cdot 6$

$-2 = d$

b. Write a recursive formula for a_n.

$$\begin{cases} a_1 = 36 \\ a_n = a_{n-1} - 2, \text{ for } n \geq 2 \end{cases}$$

c. Write an explicit formula for a_n.

$a_n = 36 + -2(n-1)$

d. Find a_{32}.

$a_{32} = 36 + -2(32-1)$

$= 36 + -2 \cdot 31$

$= -26$

9. $a_4 = 35$, $a_9 = 75$

a. Find d.

$75 = a_1 + d(9-1)$

$35 = a_1 + d(4-1)$

$$\begin{cases} 75 = a_1 + 8d \\ -35 = {}^-a_1 + {}^-3d \end{cases}$$

$40 = 5d$

$8 = d$

b. Write a recursive formula for a_n.

$$\begin{cases} a_1 = 11 \\ a_n = a_{n-1} + 8, \text{ for } n \geq 2 \end{cases}$$

c. Write an explicit formula for a_n.

$a_n = 11 + 8(n-1)$

d. Find a_{100}.

$a_{100} = 11 + 8(100-1)$

$= 803$

Geometric sequence formulas and their derivations:

Term	8	40	200	1000	…	
Symbols	a_1	a_2	a_3	a_4	…	a_n
Term number	1	2	3	4		n
Process	$8 \cdot 5^0$	$8\bullet(5^1)$	$8 \cdot 5^2$	$8 \cdot 5^3$	…	$8 \cdot 5^{n-1}$

General explicit formula:

$a_n = a_1 \cdot r^{n-1}$

Recursive formula:

$$\begin{cases} a_1 = \text{start} \\ a_n = r \cdot a_{n-1}, \text{ for } n \geq 2 \end{cases}$$

Let a_n be a geometric sequence.

10. $a_1 = 3$, $a_2 = 15$

a. Find r.

$r = 5$

b. Write a recursive formula for a_n.

$$\begin{cases} a_1 = 3 \\ a_n = 5 \cdot a_{n-1}, \text{ for } n \geq 2 \end{cases}$$

c. Write an explicit formula for a_n.

$a_n = 3 \cdot 5^{n-1}$

d. Find a_9.

$a_9 = 3 \cdot 5^{9-1}$

$= 3 \cdot 5^8$

$= 1,171,875$

11. $a_1 = 48$, $a_7 = \dfrac{16}{243}$

a. Find r.

$\frac{16}{243} = 48 \cdot r^{7-1}$

$\left(\frac{1}{729}\right)^{1/6} = (r^6)^{1/6}$

$r = \frac{1}{3}$

b. Write a recursive formula for a_n.

$$\begin{cases} a_1 = 48 \\ a_n = \frac{1}{3} \cdot a_{n-1}, \text{ for } n \geq 2 \end{cases}$$

c. Write an explicit formula for a_n.

$a_n = 48\left(\frac{1}{3}\right)^{n-1}$

d. Find a_6.

$a_6 = \frac{16}{81}$

☆ I used the recursive formula!

12. $a_3 = 8$, $a_7 = 2,048$

a. Find r.

$8 = a_1 \cdot r^{3-1}$

$2048 = a_1 \cdot r^{7-1}$

$$\begin{cases} 8 = a_1 \cdot r^2 \rightarrow a_1 = \frac{8}{r^2} \\ 2048 = a_1 \cdot r^6 \end{cases}$$

$2048 = \frac{8}{r^2} \cdot r^6$

$2048 = 8r^4$

$r = 4$

b. Write a recursive formula for a_n.

$$\begin{cases} a_1 = 1/2 \\ a_n = 4 \cdot a_{n-1}, \text{ for } n \geq 2 \end{cases}$$

c. Write an explicit formula for a_n.

$a_n = \frac{1}{2}(4)^{n-1}$

d. Find a_{11}.

$a_{11} = \frac{1}{2}(4)^{11-1}$

$= 524,288$