11.3 Sequences and Series - Word Problems
Group Practice

You will work in groups of three to answer each of the following word problems. A different person must come up each time to check the answer cards. You must WORK TOGETHER! ☺

1) An auditorium with 30 rows of seats has 20 seats in the first row, 24 seats in the second row, 28 seats in the third row, and so on.

arithmetic: $d = 4$, $a_1 = 20$

a. How many seats in the 20th row?

$a_{20} = 20 + (20-1)4$

$a_{20} = 20 + (19)4$

$a_{20} = 96$ seats

b. Determine the maximum capacity of the auditorium.

$S_{30} = \frac{30}{2}(20 + 136)$ ← $a_{30} = 20 + (30-1)4 = 20 + 29(4) = 136$

$S_{30} = 2340$ seats

2) A county fair is holding a baked goods competition in which the top eight bakers receive cash prizes. First place receives a cash prize of \$200, second place receives \$175, third place receive \$150, and so on.

arithmetic: $d = -25$, $a_1 = 200$

a. How much does the 8th place baker receive?

$a_8 = 200 + -25(8-1)$

$a_8 = 200 + -25(7)$

$a_8 = 25$

\$25

b. What is the total amount of prize money awarded at the competition.

$S_8 = \frac{8}{2}(200 + 25)$

$S_8 = 4(225)$

$S_8 =$ \$900

3) An out of state plumbers union contract specifies that each worker will receive a 5% pay increase each year for the next 30 years. One worker is paid \$20,000 for the first year. What is this person's total lifetime salary over a 30 year period?

geometric: $r = 1.05$, $a_1 = 20{,}000$ ← careful... your first thought might be to make the rate .05

sum :)

$S_n = \frac{a_1(1-r^n)}{1-r}$

$S_{30} = \frac{20{,}000(1-1.05^{30})}{1-1.05} =$ \$1,328,777

4) This week your parents gave you a \$5 allowance, and they said as long as you keep your grades up they will increase your allowance each week by \$2. (consider \$5 to be the first week)

arithmetic: $a_1 = 5$, $d = 2$

a. How much is your allowance in the 20th week?

$a_n = a_1 + (n-1)d$

$a_{20} = 5 + (19)2$

$a_{20} = 43$

\$43 ←

b. If you saved all of your money from the first 40 weeks, how much total would you have?

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{40} = \frac{40}{2}(5 + 83)$ ← $a_{40} = 5 + (39)\cdot 2$, $a_{40} = 83$

$S_{40} = 1760$

5. *A tax rebate that returns a certain amount of money to taxpayers can have a total effect on the economy that is many times this amount. This phenomenon in economics, this is called the multiplier effect.*

Suppose that the government reduces taxes so that a consumer has $2000 more in income. The government assumes each person will spend 70% of this. The individuals and businesses receiving that money will spend 70% of that, creating extra income for other people to spend and so on *(hint: think infinite!)* Determine the total amount spent on consumer goods from the initial $2000 tax rebate.

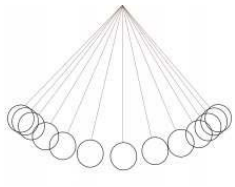
geometric: $a_1 = .7(2000) = 1400$, $r = .7$

$S_\infty = \frac{a_1}{1-r}$

$S_\infty = \frac{1400}{1-.7} = \$4667$

6. The bob of a pendulum swings through an arc 24cm long on its first swing. If each successive swing is approximately five-sixths of the length of the preceding swing, how far will the bob travel before coming to rest? *(hint: think infinite!)*

geometric: $a_1 = 24$, $r = 5/6$ think: $24 + 24(\frac{5}{6}) + 24(\frac{5}{6})^2 + \ldots$ infinite :)



$S_\infty = \frac{a_1}{1-r}$

$S_\infty = \frac{24}{1-5/6} = 144\text{ cm}$

***7. A bouncy ball is dropped from a height of 30 feet out a 3 story building and bounces 68% of its original height on each subsequent bounce.

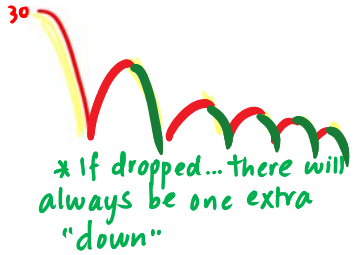
geometric: $a_1 = 20.4$, $r = .68$

a. What is the total distance that the ball travels when it hits the ground for the 6th time?

$S_n = \frac{a_1(1-r^n)}{1-r}$: S_5 downs $= \frac{20.4(1-.68^5)}{1-.68}$ + initial drop

S_5 ups $= \frac{20.4(1-.68^5)}{1-.68}$

$= 138.96$



b. What is the total distance that the ball travels before it stops?

$S_\infty = \frac{a_1}{1-r} \cdot 2 + 30$

$= 151.5$

8) A snail is crawling up a straight wall. The first hour it climbs 16 inches, the second hour it climbs 12 inches, and each succeeding hour it climbs only three-fourths the distance it climbed the previous hour. Assume the pattern continues.

geometric: $a_1 = 16$, $r = 3/4$

a. How far does the snail climb during the 7th hour? $a_n = a_1 \cdot r^{n-1}$

$a_7 = 16(\frac{3}{4})^6$

$a_7 \approx 2.85$ inches

b. What is the total distance the snail has climbed in seven hours?

$S_n = \frac{a_1(1-r^n)}{1-r}$ $S_7 = \frac{16(1-.75^7)}{1-.75} = 55.5$ inches

not an easy life :

c. Express the total distance in part c using summation notation.

$$\sum_{n=1}^{7} 16\left(\frac{3}{4}\right)^{n-1}$$